SOME APPLICATIONS OF MULTIVARIATE KRIGING IN GROUND WATER HYDROLOGY

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ILLUSTRATIONS

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ABSTRACT

Transmissivity is an important aquifer parameter in ground water hydrology, but owing to the difficulties in obtaining field data, e.g. from pumping tests, there are often few transmissivity values available. Therefore, other aquifer parameters which are in general, well correlated with the transmissivity, viz. specific capacity, electrical resistivity, etc. are used to estimate the transmissivity. The aquifer chosen for the study is a semi-confined alluvial aquifer in Tunisia, where the data on transmissivity and specific capacity are very scarce but supported by a large number of data on electrical transverse resistance. Two geostatistical methods of handling several variables together viz., cokriging and kriging with an external drift were used. The structural models showing the variability and covariability of the three variables were calculated and modeled, using the weighted least squares method. The fitted variograms and cross-variograms were then validated to suit the observed data and to determine the best combination of the inputs. During this validation, the two methods were also compared. The study shows that we can improve the estimation of aquifer transmissivity by using the other related parameters that are easily available. The problem of working on a smaller set of data, which is an important feature in applying the geostatistical methods to ground water hydrology, is also discussed.

RESUME

La transmissivitè joue un rôle très important dans l'étude des systémes aquifères. On la mesure par des expériences in situ, dont la complexitè explique le manque de donnèes disponibles. Un certain nombre d'autres paramètres, tels que

le débit spécifique et la résistivité électrique, qui sont, en général, bien correlés à la transmissivité, ont été utilisés pour estimer celle-ci. Pour cette étude, un aquifère alluvial semi-captif en Tunisie a été choisi. Pour la transmissivité et le débit spécifique, peu de valeurs sont disponibles, mais il en existe relativement plus sur la résistance électrique transversale. Deux méthodes géostatistiques d'estimation, qui sont capables de traiter ensemble plusieurs variables, le cokrigeage et le krigeage avec une dérive externe, ont été utilisées. Les variogrammes expérimentaux montrant la variabilité et la covariabilité spatiales, ont d'abord été calculés et leurs modèles ajustés par la méthode des moindres carrés pondérés. Ensuite, ces modèles ont été vérifiés en réestimant, les données disponibles afin de trouver la meilleure combinaison des données. Parallèlement, les deux méthodes ont été comparées. On trouve ainsi que l'estimation de la transmissivité peut être améliorée par l'utilisation d'autres paramètres correlés, facilement disponibles. Les difficultés d'utiliser les méthodes géostatistiques, quand les données sont insuffisantes, ce qui est souvent le cas en hydrogéologie, sont également évoquées.

A - INTRODUCTION

In present day research, geostatistics have a multitude of applications in ground water hydrology and therefore, describing all of them in detail is very difficult in one single paper. A number of authors, e.g., Delhomme (1974,76,78), Marsily et al. (1984), Marsily (1986), Marsily and Ahmed (1987), Ahmed and Marsily (1987a), Aboufirassi and Marino (1983,84), Gambolati and Volpi (1979), Kitanidis and Vomvoris (1983), Gutjahr et al. (1978) etc. have discussed the different applications of geostatistical methods to ground water problems. Some further developments in this field will be described here. More emphasis will be given on the practical aspects of how to work with the hydrogeological data which are always scarce and often inaccurately measured. Multivariate geostatistics are gaining importance in ground water hydrology due to the fact that although data on the variable of interest are always in short supply, some supplementary variables are comparatively well sampled.

In ground water hydrology two variables viz., transmissivity and piezometric head are the most important among a number of aquifer parameters. Transmissivity, which is a measure of the ease of flow of ground water, is measured by a complicated field experiment called a pumping test. The observed data are always a few in number and sporadic. A few other aquifer parameters e.g., specific capacity of the well, electrical resistivity of the formation, elastic properties etc. are, in general, related to the transmissivity and are comparatively more easily available. Therefore, they can be utilized for estimating transmissivity values either at a point or over a block. Another example is the piezometric head which is related to rainfall, pumping rates etc. Multivariate kriging techniques allow us to use different types of input jointly to estimate any of these parameters. There are a number of such methods for treating two or more variables e.g., kriging combined with linear regression, cokriging, kriging with an external drift, kriging with a guess field etc. A comparison of these methods is available in

Ahmed and Marsily (1987a). The nature of the correlation varies from variable to variable and the exact relation is unknown. Relating several correlated variables and estimating one of them by the regression technique is quite common but this technique does not consider the spatial variability of the phenomenon; it simply neglects the spatial co-ordinates of the data. Geostatistics are based on the spatial variability and covariability of the different variables. Cokriging is a geostatistical method by which several variables are used together in the estimation using their covariability. Another method of multivariate kriging, called kriging with an external drift, takes a function of the different related variables as the conditional expectation of the variable of interest.

B - DESCRIPTION OF THE AREA AND THE DATA

1 - THE AQUIFER

The studied aquifer, is situated in the upper Medjerda valley in Tunisia. The area, located in Fig.1, is about 180 Km. to the west of Tunis near the boundary with Algeria. The aquifer is a semi-confined aquifer consisting of quaternary alluvial deposits. It can be described as a leaky confined aquifer because of the interfingering of sand and clay lenses. The perennial river Medjerda flows through the area and two of its tributaries viz., Rarai and Meliz meet near the village of Chemtou. The aquifer covers an area of about 150 Km .

2 - THE INPUT DATA

Andrieu and Talbot (1972) have studied the relation between hydraulic and electrical parameters in the area using the regression method and a few pairs of available data on transmissivity and electrical transverse resistance. Transmissivity and specific capacity data, mostly from short duration pumping tests, are available at 16 wells faîrly well distributed in the area (Talbot, 1970 and 71). There is a single well where specific capacity is available but not the transmis-

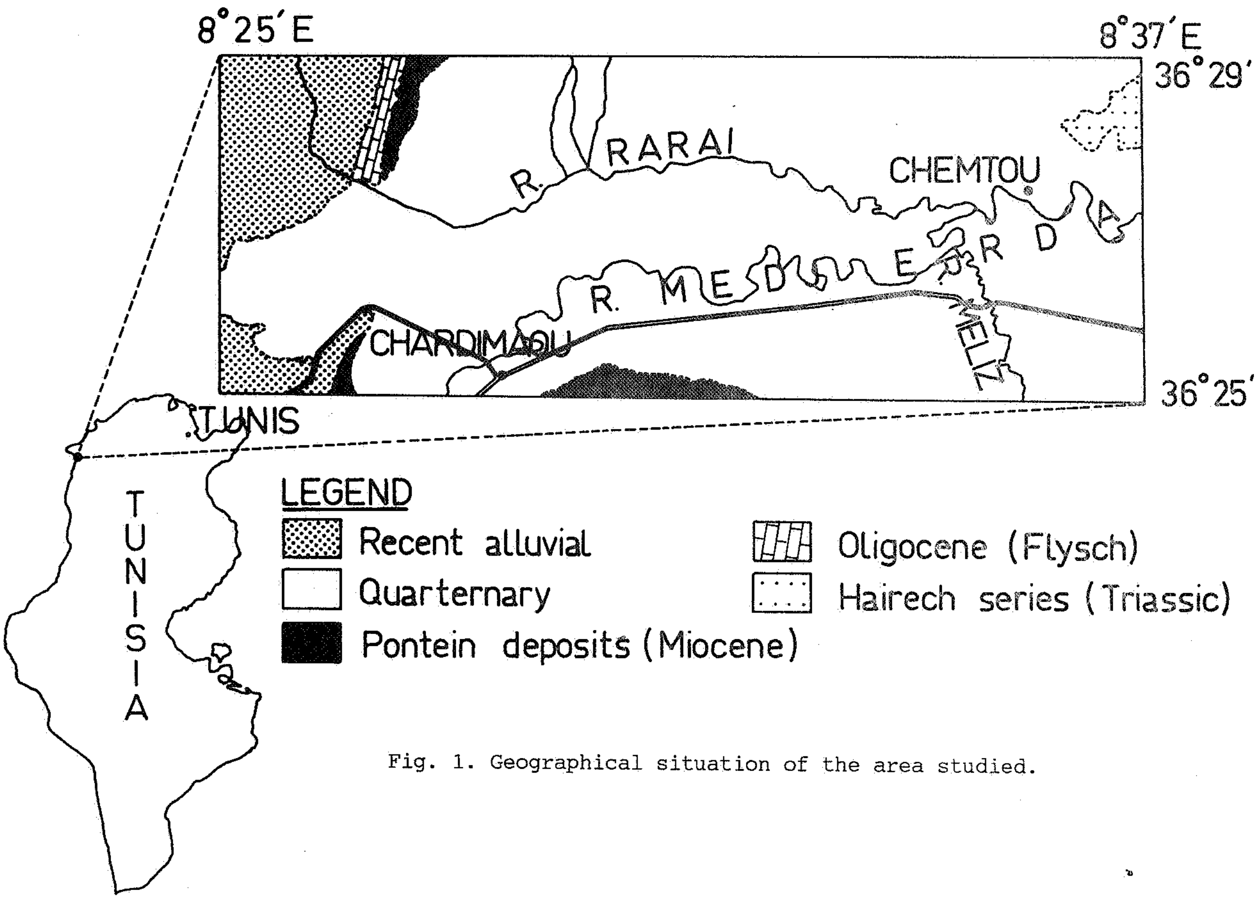


Fig. 1. Geographical situation of the area studied.

sivity. Data on transverse resistance exist at 82 points in the area. Andrieu and Talbot (1972) calculated the transverse resistance based on the apparent resistivity and saturated thickness, measured through Vertical Electrical Sounding (VES). The transverse resistance was corrected for the variations in the water resistivity measured by sampling water in the wells. The locations of different types of data are shown in Fig. 2.

From previous studies we have enough evidence that parameters such as transmissivity, specific capacity and transverse resistance show a lognormal distribution and hence they were transformed and their logarithms taken. For the sake of simplicity we will call the three parameters viz., transmissivity, specific capacity and transverse resistance as T, S and R respectively and their logarithms Z, Y and W respectively. Also the variables are represented by capital letters and their realizations by respective small letters.

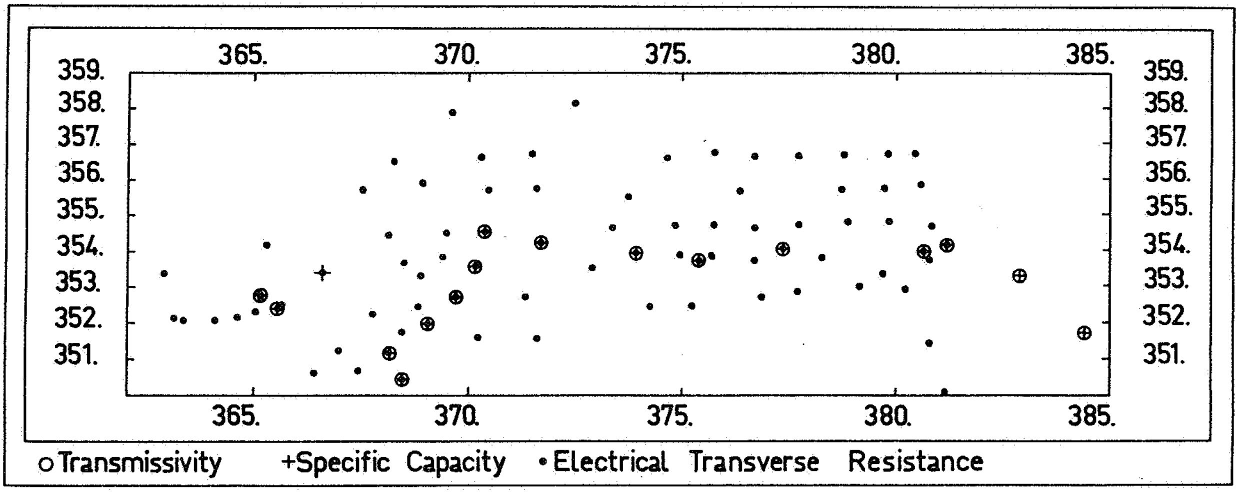


Fig. 2. Location of the input data.

C – MULTIVARIATE KRIGING ESTIMATION

1 – THEORY

The details of the theory and the developments of kriging equations will not be given here. Interested readers can refer to Matheron (1971), Journel and Huijbregts (1978), Myers(1982,83,85), Marsily (1986) for cokriging and Delhomme (1979), Galli and Meunier (1987), Ahmed and Marsily (1987a & b) for the method of kriging with an external drift. In short, we can write the system of kriging equations and the expressions for variance of the estimation error for the two methods as follows:

a) Cokriging

The estimated value of any variable $Z_r(x)$ of a group of several variables $Z_p(x)$ at a point x_o can be written:

$$Z_r^*(x_o) = \sum_{i=1}^{n_r} \lambda_i^r Z_r(x_i) + \sum_{\substack{p=1 \\ p\neq r}}^{K} \sum_{j=1}^{n_p} \lambda_j^p Z_p(x_j) \qquad (1)$$

where λ's can be calculated from the following system of equations:

$$\sum_{p=1}^{K} \sum_{j=1}^{n_p} \lambda_j^p \gamma_{pq}(x_i,x_j) + \mu_q = \gamma_{rq}(x_o,x_i) \qquad (2)$$

$$q = 1,\ldots, K \qquad i = 1,\ldots, n_q$$

$$\sum_{i=1}^{n_r} \lambda_i^r = 1 \qquad (3)$$

$$\sum_{i=1}^{n_p} \lambda_i^p = 0 \qquad \forall\, p \neq r \qquad (4)$$

where

$$\gamma_{pq}(x_i,x_j) = \frac{1}{2} E \{ [Z_p(x_j)-Z_p(x_i)][Z_q(x_j)-Z_q(x_i)] \} \qquad (5)$$

is the simple variogram of the variable Z_p if q=p, otherwise the cross-variogram of the variables Z_p and Z_q. μ's are Lagrange multipliers.

The variance of the estimation error is:

$$\sigma^2(x_o) = \sum_{p=1}^{K} \sum_{i=1}^{n_p} \lambda_i^p \gamma_{rp}(x_o-x_i) + \mu_r \qquad (6)$$

For this particular case K is equal to 3, Z_1 is simply Z, Z_2 is Y and Z_3 is W.

b) Kriging with external drift

The estimator of z in this method can be written as usual,

$$Z^*(x_o) = \sum_{i=1}^{n} \lambda_i Z(x_i) \qquad (7)$$

where λ's can be calculated from the following system of equations:

$$\sum_{j=1}^{n} \lambda_j \gamma(x_i,x_j) + \mu_o + \sum_{\alpha=1}^{K} \mu_\alpha Y_\alpha(x_i) = \gamma(x_i,x_o) \qquad i = 1,\ldots, n \tag{8}$$

$$\sum_{i=1}^{n} \lambda_i = 1 \tag{9}$$

$$\sum_{i=1}^{n} \lambda_i Y_\alpha(x_i) = Y_\alpha(x_o) \qquad \alpha = 1,\ldots, K \tag{10}$$

where $\gamma(x_i,x_j) = \frac{1}{2} E\{[Z(x_j)-Z(x_i)]^2\}$ (11)

and the variance of the estimation error is:

$$\sigma^2(x_o) = \sum_{i=1}^{n} \lambda_i \gamma(x_i,x_o) + \mu_o + \sum_{\alpha=1}^{K} \mu_\alpha Y_\alpha(x_o) \tag{12}$$

μ_o and μ_α are Lagrange multipliers. In this case K is equal to 2, Y_1 is simply Y and Y_2 is W.

c) Structural models and their validation

An important point to be noted is that the calculation of the variogram/cross-variogram makes a few approximations such as using a tolerance on distance or on angle (in case of directional variograms) to have more pairs, replacing the expectation by arithmetic mean etc. Another approximation is made by fitting a simple mathematical function as the theoretical variogram. Then the variogram/cross-variogram may or may not represent the true variability of the phenomenon and hence it has to be verified against some known base i.e. the observed data. So each observed value is estimated using the remaining data and the selected variogram model. This is called the cross-validation and the models are modified and finally chosen on the basis of the following results:

$$(1/n) \sum_{i=1}^{n} (Z_i^* - Z_i) \simeq 0 \tag{13}$$

$$(1/n) \sum_{i=1}^{n} (Z_i^* - Z_i)^2 \simeq \min \tag{14}$$

$$(1/n) \sum_{i=1}^{n} \{(Z_i^* - Z_i)/\sigma_i\} \simeq 0.0 \tag{15}$$

$$(1/n) \sum_{i=1}^{n} \{(Z_i^* - Z_i)^2/\sigma_i^2\} \simeq 1.0 \tag{16}$$

where z_i^* and z_i are the estimated and the observed values of the variable at the location i, σ_i is the associated standard deviation of the estimation error.

The cross-variogram should satisfy the following constrains in order to have a positive definite variogram matrix:

$$\gamma_{pq} \leq \sqrt{\gamma_{pp}\gamma_{qq}} \tag{17}$$

which in case, the variogram types are same, can be simplified to:

$$c_{pq} \leq \sqrt{c_{pp}c_{qq}} \tag{18}$$

$$a_{pq} \geq \sqrt{a_{pp}a_{qq}} \tag{19}$$

where c and a are the sill and the range of the corresponding variograms.

The above conditions are necessary but not sufficient to check that the variogram matrix is positive definite. Another verification of this criterion can be made as follows:

$$\gamma_{p+q} = \gamma_{pp} + \gamma_{qq} + 2\gamma_{pq} \tag{20}$$

$$\gamma_{p-q} = \gamma_{pp} + \gamma_{qq} - 2\gamma_{pq} \tag{21}$$

where γ_{p+q} and γ_{p-q} are the variograms of the sum and difference of the two variables Z_p and Z_q respectively. The details of these conditions are given in Myers (1982).

2 - ESTIMATION

The three variables are very well correlated as shown by the follwing regression relations (Fig. 3).

$Y = -0.0862 + 0.8533\ Z$	$r=0.9449$	(22)
$W = 3.4270 + 0.2500\ Z$	$r=0.8821$	(23)
$W = 3.5442 + 0.2397\ Y$	$r=0.8923$	(24)

where r means the coefficient of correlation.

Fig. 3(a)

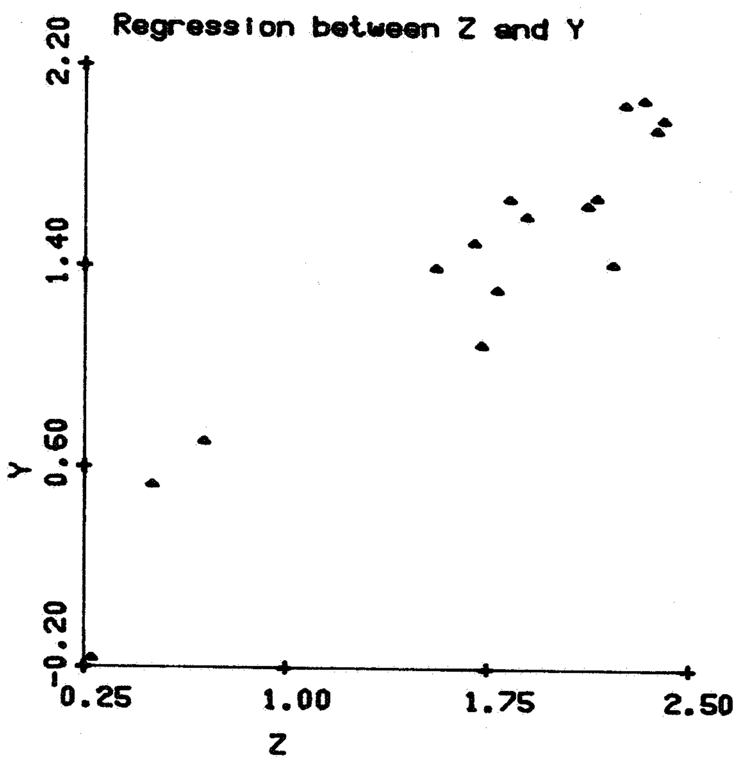


Fig. 3(b) **Regression between Z and W**

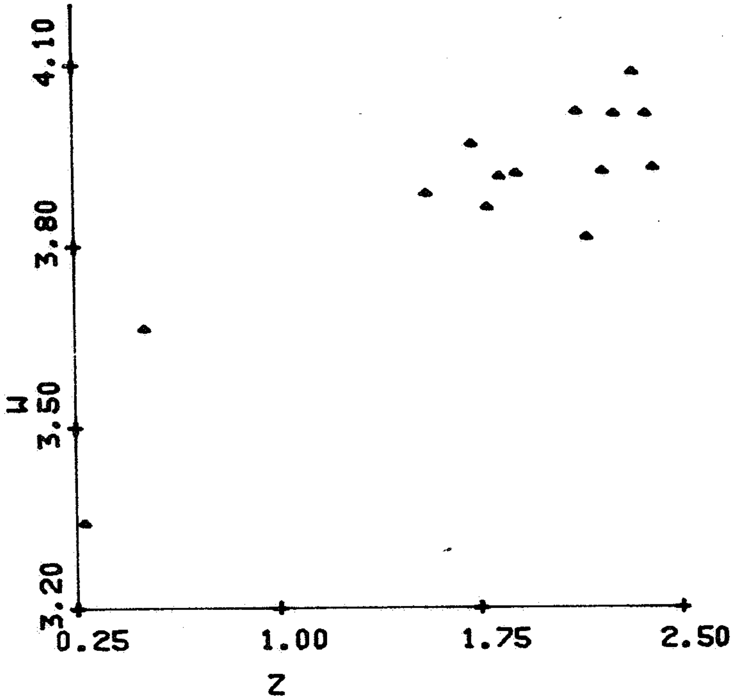
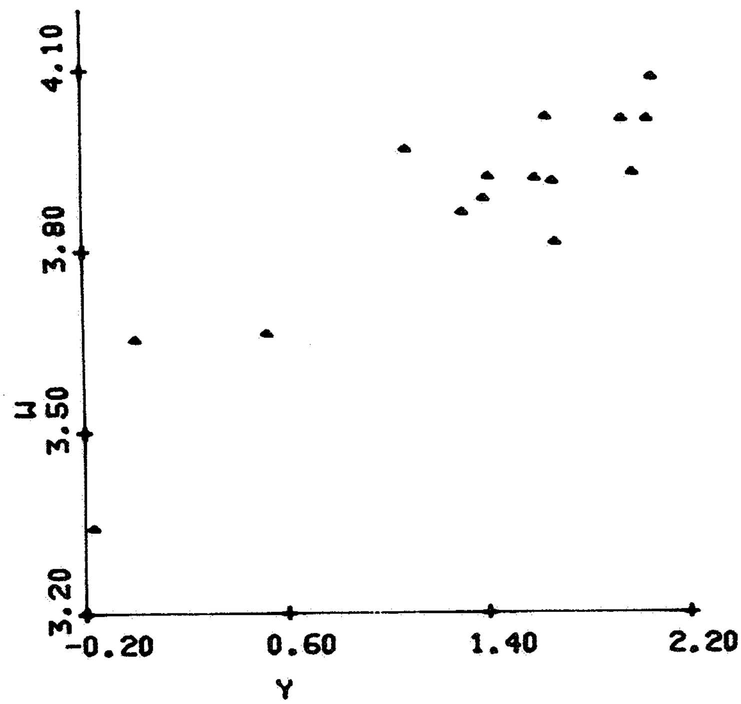
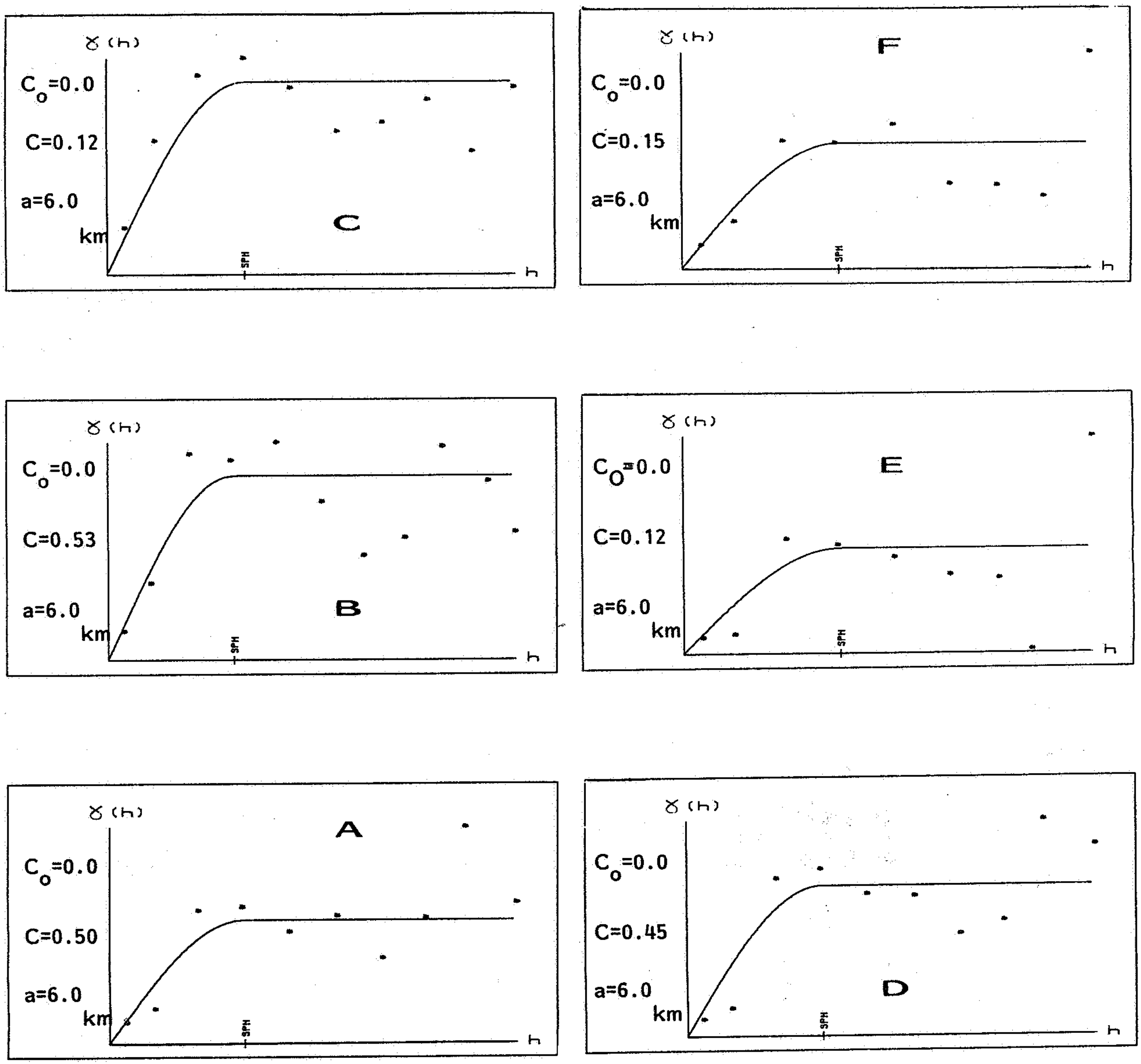


Fig. 3(c) **Regression between Y and W**





C_o - nugget effect   C - sill   a - range

Fig. 4 (A) Variogram of Z
(B) Variogram of Y
(C) Variogram of W
(D) Cross-variogram of Z and Y
(E) Cross-variogram of Z and W
(F) Cross-variogram of Y and W

The variograms and the cross-variograms were calculated from the available data and fitted to simple mathematical functions (Fig. 4). The variogram models were validated by slightly varying their parameters (model type, sill and range). First of all it was verified that the three simple variograms reproduced the corresponding field values by using the method of ordinary kriging (see tables I, II, III). The best model was chosen on the basis of the results given by equations 13 to 16. To validate the cross-variograms the method of cokriging was used to reproduce the data on log-T. The cross-variogram of Z and Y was validated

Table I: Cross-validation for γ_{ZZ} using ordinary kriging with log-T data

Variogram model (γ_{ZZ})			values of equation			
type	sill	range	13	14	15	16
sph	0.45	5.00	0.0085	0.1557	0.0042	0.7238
sph	0.45	6.00	0.0134	0.1531	0.0099	0.8433
sph	0.45	7.00	0.0196	0.1493	0.0180	0.9567
sph	0.50	5.00	0.0085	0.1557	0.0040	0.6514
sph	0.50	6.00	0.0134	0.1531	0.0094	0.7589
sph	0.50	7.00	0.0196	0.1493	0.0171	0.8610
sph	0.55	5.00	0.0085	0.1557	0.0038	0.5922
sph	0.55	6.00	0.0134	0.1531	0.0090	0.6899
sph	0.55	7.00	0.0196	0.1493	0.0163	0.7827
lin	0.45	5.00	0.0374	0.1510	0.0375	0.9704
lin	0.45	6.00	0.0374	0.1509	0.0411	1.1645
lin	0.45	7.00	0.0374	0.1511	0.0444	1.3586
lin	0.50	5.00	0.0374	0.1510	0.0356	0.8734
lin	0.50	6.00	0.0374	0.1509	0.0390	1.0481
lin	0.50	7.00	0.0374	0.1511	0.0421	1.2228
lin	0.55	5.00	0.0374	0.1510	0.0339	0.7940
lin	0.55	6.00	0.0374	0.1509	0.0372	0.9528
lin	0.55	7.00	0.0374	0.1511	0.0402	1.1116
exp	0.45	5.00	0.0207	0.1434	0.0211	1.0488
exp	0.45	6.00	0.0232	0.1431	0.0260	1.2320
exp	0.45	7.00	0.0251	0.1431	0.0303	1.4176
exp	0.50	5.00	0.0207	0.1434	0.0200	0.9439
exp	0.50	6.00	0.0232	0.1431	0.0247	1.1088
exp	0.50	7.00	0.0251	0.1431	0.0288	1.2759
exp	0.55	5.00	0.0207	0.1434	0.0191	0.8581
exp	0.55	6.00 *	0.0232	0.1431	0.0235	1.0080
exp	0.55	7.00	0.0251	0.1431	0.0274	1.1599
sph- spherical, exp- exponential, lin- linear						

* The variogram taken finally

using the data on Z and Y and similarly the cross-variogram of Z and W was validated using data on Z and W. The cross-variogram of Y and W was validated using data on Z, Y and W. In all the cases the field values of Z were estimated for comparison. The coefficients of the cross-variograms were slightly modified and the different results of validations for the cross-variograms are given in table IV, V and VI. Care has been taken to satisfy the conditions given in equations 17 to 19 while changing the variogram parameters during the cross-validation.

Table II: Cross-validation for γ_{YY} using ordinary kriging with log-S data

Variogram model (γ_{YY})			values of equation			
type	sill	range	13	14	15	16
sph	0.48	5.00	0.0159	0.1840	0.0161	0.9185
sph	0.48	6.00	0.0194	0.1816	0.0213	1.0814
sph	0.48	7.00	0.0252	0.1668	0.0296	1.1752
sph	0.53	5.00	0.0159	0.1840	0.0153	0.8319
sph	0.53	6.00	0.0194	0.1816	0.0202	0.9794
sph	0.53	7.00	0.0252	0.1668	0.0281	1.0644
sph	0.60	5.00	0.0159	0.1840	0.0144	0.7348
sph	0.60	6.00	0.0194	0.1816	0.0190	0.8651
sph	0.60	7.00	0.0252	0.1668	0.0265	0.9402
lin	0.48	5.00	0.0436	0.1711	0.0507	1.2311
lin	0.48	6.00	0.0436	0.1710	0.0556	1.4773
lin	0.48	7.00	0.0436	0.1712	0.0600	1.7236
lin	0.53	5.00	0.0436	0.1711	0.0483	1.1150
lin	0.53	6.00	0.0436	0.1710	0.0529	1.3380
lin	0.53	7.00	0.0436	0.1712	0.0571	1.5610
lin	0.60	5.00	0.0436	0.1711	0.0454	0.9849
lin	0.60	6.00	0.0436	0.1710	0.0497	1.1819
lin	0.60	7.00	0.0436	0.1712	0.0537	1.3789
exp	0.48	5.00	0.0265	0.1667	0.0333	1.3368
exp	0.48	6.00	0.0290	0.1655	0.0394	1.5698
exp	0.48	7.00	0.0308	0.1649	0.0448	1.8055
exp	0.53	5.00	0.0265	0.1667	0.0317	1.2107
exp	0.53	6.00	0.0290	0.1655	0.0375	1.4217
exp	0.53	7.00	0.0308	0.1649	0.0427	1.6352
exp	0.60	5.00	0.0265	0.1667	0.0298	1.0694
exp	0.60	6.00 *	0.0290	0.1655	0.0352	1.2559
exp	0.60	7.00	0.0308	0.1649	0.0401	1.4444
sph- spherical, exp- exponential, lin- linear						

* The variogram taken finally

Table III: Cross-validation for γ_{WW} using ordinary kriging with log-R data

Variogram model (γ_{WW})			values of equation			
type	sill	range	13	14	15	16
sph	0.12	5.00	0.0127	0.0259	0.0515	0.8984
sph	0.12	6.00	0.0098	0.0256	0.0350	1.0258
sph	0.12	7.00	0.0107	0.0284	0.0367	1.2648
sph	0.10	5.00	0.0127	0.0259	0.0564	1.0781
sph	0.10	6.00	0.0098	0.0256	0.0383	1.2310
sph	0.10	7.00	0.0107	0.0284	0.0402	1.5178
sph	0.14	5.00	0.0127	0.0259	0.0477	0.7701
sph	0.14	6.00	0.0098	0.0256	0.0324	0.8793
sph	0.14	7.00	0.0107	0.0284	0.0340	1.0841
lin	0.12	5.00	0.0087	0.0259	0.0466	1.3682
lin	0.12	6.00	0.0067	0.0258	0.0286	1.5614
lin	0.12	7.00	0.0107	0.0295	0.0479	1.9658
lin	0.10	5.00	0.0087	0.0259	0.0511	1.6418
lin	0.10	6.00	0.0067	0.0258	0.0313	1.8737
lin	0.10	7.00	0.0107	0.0295	0.0524	2.3590
lin	0.14	5.00	0.0087	0.0259	0.0432	1.1727
lin	0.14	6.00	0.0067	0.0258	0.0265	1.3384
lin	0.14	7.00	0.0107	0.0295	0.0443	1.6850
exp	0.10	5.00	0.0101	0.0256	0.0571	1.6484
exp	0.10	6.00	0.0078	0.0253	0.0364	1.8722
exp	0.10	7.00	0.0116	0.0290	0.0574	2.3527
exp	0.12	5.00	0.0101	0.0256	0.0521	1.3737
exp	0.12	6.00	0.0078	0.0253	0.0332	1.5602
exp	0.12	7.00	0.0116	0.0290	0.0524	1.9606
exp	0.14	5.00	0.0101	0.0256	0.0483	1.1774
exp	0.14	6.00 *	0.0078	0.0253	0.0308	1.3373
exp	0.14	7.00	0.0116	0.0290	0.0485	1.6805
sph- spherical, exp- exponential, lin- linear						

* The variogram taken finally.

Table IV cross-validation for γ_{ZY} using cokriging with log-T and log-S data

p	q	type	sill	range		13	14	15	16
1	1	exp	0.55	6.00					
2	1	exp	0.45	6.00					
2	2	exp	0.60	6.00		0.0170	0.0722	0.0306	1.7835
1	1	exp	0.55	6.00					
2	1	exp	0.50	6.00					
2	2	exp	0.60	6.00		0.0165	0.0745	0.0392	3.0599
1	1	exp	0.55	6.00					
2	1	exp	0.40	6.00					
2	2	exp	0.60	6.00		0.0174	0.0720	0.0263	1.2693
1	1	exp	0.55	6.00					
2	1	exp	0.45	7.00	*				
2	2	exp	0.60	6.00		0.0180	0.0713	0.0297	1.1845
1	1	exp	0.55	6.00					
2	1	exp	0.50	7.00					
2	2	exp	0.60	6.00		0.0185	0.0710	0.0382	1.5296
1	1	exp	0.55	6.00					
2	1	exp	0.40	7.00					
2	2	exp	0.60	6.00		0.0181	0.0732	0.0256	0.9890

Column groups: p–range = variogram/cross-variogram; 13–16 = values of equation.

* The cross-variogram taken finally

Table V cross-validation for γ_{ZW} using cokriging with log-T and log-R data

variogram/cross-variogram					values of equation			
p	q	type	sill	range	13	14	15	16
1	1	exp	0.55	6.00				
2	1	exp	0.08	6.00				
2	2	exp	0.14	6.00				
********					0.0258	0.1255	0.0316	0.8778
1	1	exp	0.55	6.00				
2	1	exp	0.12	6.00				
2	2	exp	0.14	6.00				
********					0.0280	0.1205	0.0389	0.8947
1	1	exp	0.55	6.00				
2	1	exp	0.16	6.00 *				
2	2	exp	0.14	6.00				
********					0.0307	0.1180	0.0493	0.9969
1	1	exp	0.55	6.00				
2	1	exp	0.20	6.00				
2	2	exp	0.14	6.00				
********					0.0335	0.1178	0.0653	1.2873
1	1	exp	0.55	6.00				
2	1	exp	0.08	7.00				
2	2	exp	0.14	6.00				
********					0.0229	0.1279	0.0233	0.8910
1	1	exp	0.55	6.00				
2	1	exp	0.12	7.00				
2	2	exp	0.14	6.00				
********					0.0236	0.1231	0.0254	0.8915
1	1	exp	0.55	6.00				
2	1	exp	0.16	7.00				
2	2	exp	0.14	6.00				
********					0.0247	0.1201	0.0293	0.9431
1	1	exp	0.55	6.00				
2	1	exp	0.20	7.00				
2	2	exp	0.14	6.00				
********					0.0264	0.1187	0.0359	1.0789

* The cross-variogram taken finally

Table VI cross-validation for γ_{YW} using cokriging with log-T, log-S and log-R data

variogram/cross-variogram					values of equation			
p	q	type	sill	range	13	14	15	16
1	1	exp	0.55	6.00				
2	1	exp	0.45	7.00				
3	1	exp	0.16	6.00				
2	2	exp	0.60	6.00				
3	2	exp	0.20	6.00 *				
3	3	exp	0.14	6.00				
********					0.0178	0.0673	0.0302	1.1587
1	1	exp	0.55	6.00				
2	1	exp	0.45	7.00				
3	1	exp	0.16	6.00				
2	2	exp	0.60	6.00				
3	2	exp	0.15	6.00				
3	3	exp	0.14	6.00				
********					0.0188	0.0685	0.0347	1.2954
1	1	exp	0.55	6.00				
2	1	exp	0.45	7.00				
3	1	exp	0.16	6.00				
2	2	exp	0.60	6.00				
3	2	exp	0.10	6.00				
3	3	exp	0.14	6.00				
********					0.0200	0.0723	0.0402	1.5840
1	1	exp	0.55	6.00				
2	1	exp	0.45	7.00				
3	1	exp	0.16	6.00				
2	2	exp	0.60	6.00				
3	2	exp	0.20	7.00				
3	3	exp	0.14	6.00				
********					0.0183	0.0677	0.0325	1.2213
1	1	exp	0.55	6.00				
2	1	exp	0.45	7.00				
3	1	exp	0.16	6.00				
2	2	exp	0.60	6.00				
3	2	exp	0.15	7.00				
3	3	exp	0.14	6.00				
********					0.0192	0.0698	0.0366	1.3929
1	1	exp	0.55	6.00				
2	1	exp	0.45	7.00				
3	1	exp	0.16	6.00				
2	2	exp	0.60	6.00				
3	2	exp	0.10	7.00				
3	3	exp	0.14	6.00				
********					0.0202	0.0739	0.0415	1.7113

* The cross-variogram taken finally

A cross-validation test was also performed using the method of kriging with an external drift. The results with different input combinations are given below in table VII.

Table VII

The external drift used as a function of	Values of equation 13	14	15	16
log-S alone	0.0131	0.0873	0.0121	0.7536
log-R alone	0.0023	0.1619	-0.0001	0.9961
log-S & log-R both	-0.0085	0.1459	-0.0146	0.9514

Thus, the method, the input combination and the structural models giving the best results were used in estimating the Z values on a regular 1 km square grid. Figures 5 and 6 show the estimated values and the corresponding variance of the estimation error, respectively.

D – RESULTS AND DISCUSSION

As shown in tables I to VI, the structural models were changed by varying their parameters in all possible ways. It is also clear that by varying the range or the model type of the variogram one can change the estimated values. This means that a change in sill value hardly changes the estimated values but modifies the variance of the estimation error. The value given by equation 14, which gives the variance of the difference between the estimated and the true values, is a measure of the absolute error. The smaller the value of this equation the closer the estimated values are to the observed ones. So to decide which model is best, the case where equation 14 has the smallest value is taken. If, for example, the values of this equation are identical in several cases then, we compare the values given by other equations (16,13 and 15). A low value of equation 14 and a value close to 1 for equation 16, shows that the values of the estimation error are also correct together with the better estimates. Note that any variogram model giving a small value of equation 14 but a value of equation 16 far from 1, may not be acceptable. When the variogram models were validated by ordinary kriging, no case was found where the linear model provided an encouraging result. The model types for the cross-variograms were kept the same as for the simple variograms in order to make it easy to check the positive definite conditions. Different cross-variograms were validated using the corresponding combination of the input data. In the case of cokriging the validation was always performed by estimating the log-T values. The other variables used as input to cokriging could also be estimated for cross-validation. However, our main objective was to find the structural model and input combination which could reproduce the log-T values. From the four cases of log-T estimation (by ordinary kriging using Z alone, cokriging using Z and Y, cokriging using Z and W and cokriging using Z, Y and W), it can be seen that the best estimates are given when all three variables are utilized. If we compare the two methods (comparing tables IV,V and VI with the three rows of table VII respectively), we find that the best method of cokriging gives better results for all the combinations. Therefore, with the help of cross-validation one can remove the ambiguity in the structural model and choose the best combination of input data and the most suitable method.

Figure 5: Cokriging estimates of log-T

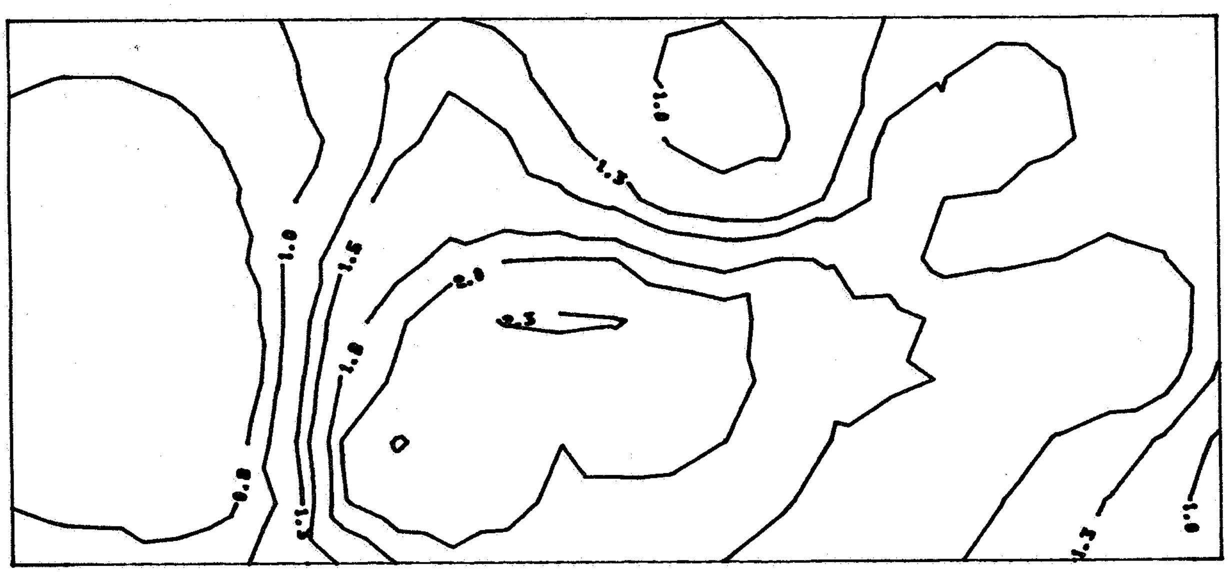
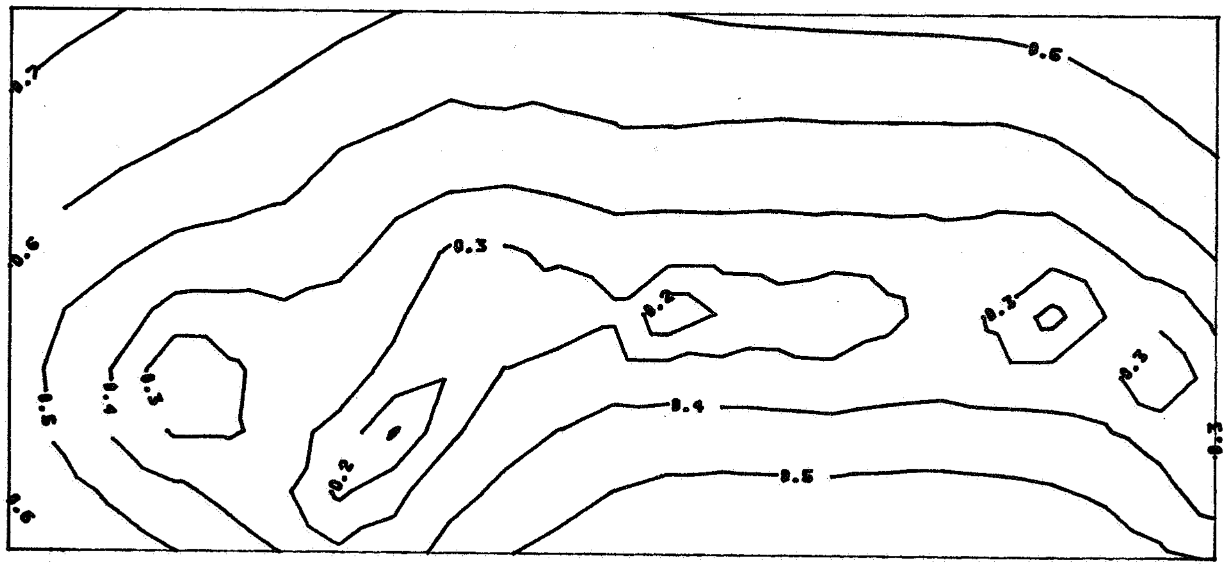


Figure 6: Satandard deviation of the estimation error



E - CONCLUSION

The main objective of this study is to show how one can apply the two methods of multivariate geostatistcs to the problems of ground water hydrology. Unlike such fields as mineral exploration etc., where a large number of data are available for use in the structural analysis of the spatial variability, we always lack information on the hydrogeological parameters, particularly on aquifer transmissivity. The present example shows two very important and interesting features of applying geostatistical methods to ground water hydrology: (1) The variable may be estimated using all the related information available. For example, in the present case the transmissivity was estimated using the data on specific capacity and electrical transeverse resistance. This additional information is, in general easily available and improves the estimation. (2) The calculated structural models viz. the experimental variogram or covariance and theoretical model fitted to them are always approximate and ambigous. Thus, it is necessary to perform the cross-validation to remove the ambiguity in the structural model and to find a better combination of the input data and the most suitable method. At the same time, we can sort the observed data and find the unreliable values or the data falling outside the range of variability.

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