LOCAL ESTIMATION OF THE RECOVERABLE RESERVES: COMPARING VARIOUS METHODS WITH THE REALITY ON A PORPHYRY COPPER DEPOSIT

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ABSTRACT

The objective of this article is to compare the estimates based on several non-linear methods with the actual figures. The methods tested were disjunctive kriging, multi-Gaussian kriging and a new method (uniform conditioning) which is simpler and computationally quicker than these two and yet still gives comparable results.

INTRODUCTION

The aim of this paper is to apply different non-linear estimation methods under similar conditions where there are enough data to make comparisons and to draw conclusions on the behavior of these methods and their constraints. The methods used are disjunctive kriging, multi-Gaussian and uniform conditioning which is being put into practice for the first time. In addition, the change of support and the bigaussian hypothesis have also been tested, but less thoroughly.

The general problem comes from the fact that the characteristics of many deposits are incompatible with the requirements of non-linear methods (i.e. strictly stationary hypothesis). For instance, in the case of porphyry copper, there is a high grade zone which often leads to preferential sampling at the detriment of the border which is poorer. We are thus confronted with a double problem: presence of a large scale drift and an irregular preferentially sampled grid. We know that these circumstances do not have a great effect on kriging with a univer-

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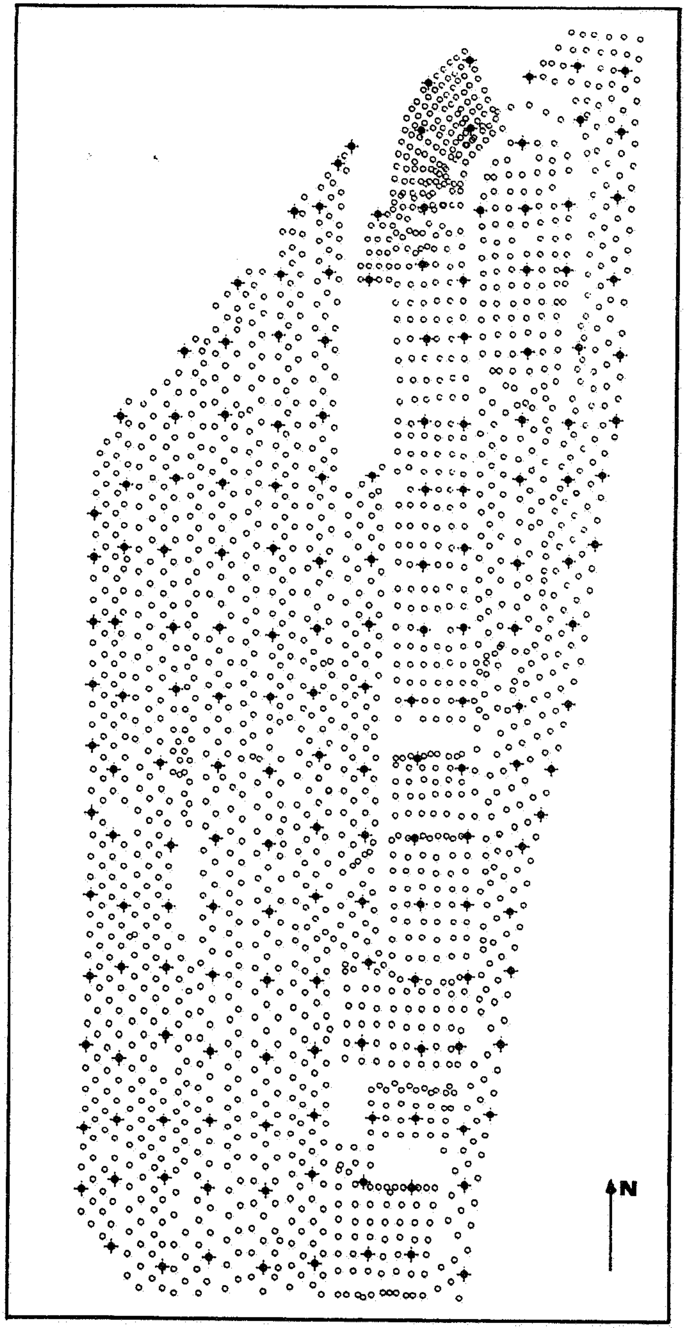


Figure 1. Location of blast-holes and of the data points seleeted.

sality condition. The estimated grades reflect the local means fairly accurately. In this case, the global mean of the kriged grades is close to the mean grade of the data weighted (by the zone of influence or by the kriging weights) and not to their raw mean grade.

The problems with non-linear methods are therefore very delicate. Theoretically the stationary hypothesis plays a fundamental part (can a drift be taken into account?). From a practical point of view, the anamorphosis that precedes almost any of these methods requires the histogram of the data to be representative.

PRELIMINARY CONDITIONS

The data come from one level of a mine where the analysed copper grades of blast-holes were available. They are considered to represent the reality. More precisely, the data correspond to a bench 13 m thick in a porphyry copper type deposit, which is thus reduced to 2 dimensions. The presence of different zones can be observed (in our case, slightly higher grades towards the south of the bench), which confirms the type of deposit mentioned above.

Results from 2 095 blast-holes are available. In order to have basic data for the estimation, these have been classified on a regular grid of panels 30 m x 30 m. By taking the most central samples on each panel, 173 blast-holes were obtained. In order to have the reference "reality", a grid of blocks 10 x 10 x 13 m (= selection unit) has been defined for the 2 095 data. In this grid, blocks with at least 2 blast-holes were taken into account. Then the mean for these blocks was computed, and they were regrouped so as to form panels 30 x 30 x 13 m . Only panels containing more than 5 blocks containing data values were taken into account.

We then have two files: one contains the basic data, the second one the "reality". The basic data are used to compute the recovery functions of blocks 10 x 10 x 13 m using the method mentioned below and the results are then compared to the contents of the second file. We have obtained 173 basic data, 583 blocks and 50 panels with 281 blocks.

REVIEW OF THE MAIN THEORETICAL POINTS

As the theory of non-linear geostatistics has been presented elsewhere (Matheron (1975 (a) (b), 1978)). Only the main points need be mentioned here.

Let Y_v denote the anamorphosed grades of blocks, v, equal in size to the selection unit. The essential problem is to estimate $Q = \Sigma f(Y_{v_i})/L$ where f(.) is a function to be estimated (e.g. the recoverable tonnage or the recoverable metal tonnage) and L is the number of blocks per panel. In cases where the number of blocks per panel is very large, L denotes the number of blocks which are used to represent the panel. These have to be selected to provide a uniform discretization of the panel.

We suppose that the decomposition of f(y) into Hermite polynomials is already known:

$$f(y) = \Sigma \frac{f_n}{n!} H_n(y) \tag{1}$$

Our objective is to estimate $\sum_{i=1}^{L} H_n(Y_{v_i})/L$ using three different sets of assumptions.

a) Multigaussian.

In this model, it is assumed that the distribution of the Y_α (point values) and of the Y_{v_i} is jointly multigaussian. The kriged estimate $Y^k_{v_i}$ (with known mean) of the block v_i represents the conditional expectation $E(Y_{v_i}|Y = y_\alpha)$; it has a standard deviation s_{v_i}.

From the classical relation

$$E[H_n(Y)|X] = \rho^n H_n(X)$$

valid for the bigaussian variables (X,Y), with correlation coefficient ρ, it is easily shown that the multigaussian estimator (MG) of $H_n(Y_{v_i})$ is given by:

$$\begin{aligned}[H_n(Y_{v_i})]_{MG} &= E[H_n(Y_{v_i})|Y] \\ &= E[H_n(Y_{v_i})|Y^k_{v_i}/s_{v_i}] \\ &= s^n_{v_i} H_n(Y^k_{v_i}/s_{v_i})\end{aligned}$$

As a matter of fact, the correlation coefficient between Y_{v_i} and $Y^k_{v_i}/s$ is equal to s_{v_i}. Therefore the MG estimator of Q can be written as:

$$f_o - f_1 \frac{1}{L} \sum_{i=1}^{L} Y^k_{v_i} + \sum_{n\geq 2} \frac{f_n}{n!} \frac{1}{L} \sum_{i=1}^{L} s^n_{v_i} H_n(Y^k_{v_i}/s_{v_i})$$

To get this estimator, L different kriging systems have to be solved.

b) Uniform Conditioning.

To avoid having to solve this number of systems, we use a single linear combination $Y^* = \sum_\alpha \lambda_\alpha Y_\alpha$ for all the v_i within a panel instead of L different ones $Y^k_{v_i}$. We actually take $Y^* = Y^k$, which is ordinary kriging of $1/L\ \Sigma Y_{v_i}$ from the Y_α. Let $Y = Y^k/s$, where s is the standard deviation of Y^k and let ρ_{v_i} the correlation coefficient between Y_{v_i} and Y^k/s (this coefficient can be calculated within the frame of the discrete Gaussian model).

The uniform conditioning (UC) estimator of $H_n(Y_{v_i})$ is

$$\begin{aligned}[H_n(Y_{v_i})]_{UC} &= E[H_n(Y_{v_i})|Y] \\ &= \rho^n_{v_i} H_n(Y^k/s)\end{aligned}$$

Expanding this, we obtain the UC estimator of Q

$$f_o - f_1 \sum_\alpha \lambda_\alpha Y_\alpha + \sum_{n\geq 2} \frac{f_n}{n!} \frac{1}{L} \sum_{i=1} \rho^n_{v_i} H_n(Y^k/s)$$

c) Disjunctive Kriging.

In this case, the disjunctive kriging estimator (DK) of $\sum_{i=1}^{L} H_n(Y_{v_i})/L$ is:

$$\sum_\alpha \lambda_{\alpha,n} H_n(Y_\alpha)$$

The coefficients $\lambda_{\alpha,n}$ are obtained by solving the system

$$\lambda_{\alpha,n} \rho^n_{\alpha\beta} = \frac{1}{L} \sum_{i=1} \rho^n_{\alpha v_i}$$

Then, the DK estimator can be expressed in the form:

$$f_o - f_1 \sum_\alpha \lambda_\alpha Y_\alpha + \sum_{n\geq 2} \frac{f_n}{n!} \sum \lambda_{\alpha,n} H_n(Y_\alpha)$$

We note that for n = 1 all the methods give the same results, which is very interesting in view of the importance of the left-hand term of the development of (1).

In practice, we calculate the point distribution in the panel and afterwards we introduce the coefficient of change of support r. That is, we calculate the equivalent of $\lambda'_\alpha = \lambda_\alpha / r$ so as to have $H_n^* = 1/L\ \Sigma\ H_n(Y_{x_i})$ for the points, and H_n^*/r^n for the blocks. This makes it possible to easily compute the recovery functions for several supports, if the discretization of the panel can be taken as constant. In terms of grade estimation, it can easily be seen that estimating a panel as:

$$Z_V = 1/L\ \Sigma\ \phi(Y_{x_i}) = \sum_n \frac{C_n}{n!} \left[1/L\ \Sigma\ H_n(Y_{x_i})\right]$$

or

$$Z_V = 1/L\ \Sigma\ \phi_r(Y_{v_i}) = \sum_n \frac{C_n}{n!} r^n \left[1/L\ \Sigma\ H_n(Y_{v_i})\right]$$

which comes back to the same thing.

HISTOGRAMS

Only main histograms obtained are presented here. Figure 2-a shows the histogram of the 173 data together with the fitted distribution and Figure 2-b shows the histogram obtained assuming the permanence of the distribution, i.e. if C_n are the coefficients fitting the point histogram, we calculate $C_n(v) = C_n r^n$ the coefficients of block histograms (10 x 10), from the experimental block histogram.

The table shows the main statistical parameters.

TABLE 1

	ACTUAL	BLOCS	Z	PANELS	BLOCS*
NUMBER OF DATA	2095	583	173	50	281
MEAN	2.315	2.321	2.304	2.453	2.450
s^2	2.755	2.155	2.657	1.713	2.437

The coefficient r of change of support is equal to 0.911 and makes it possible to go from the variance of Z to the block variance. This variance corresponds to the mean value of the covariance in the block $\overline{C}(10,10) = 2.123$ which is fairly close to the experimental variances of the blocks (column 2 of table 1) but rather different from the variance of the blocks used for the comparisons (column 5 of table 1).

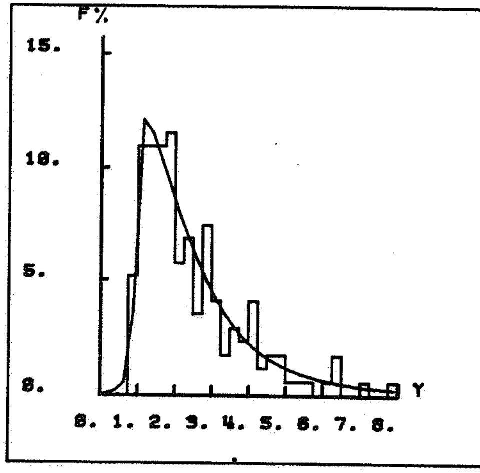


Figure 2a. Experimental histogram (173) and anamorphosis model.

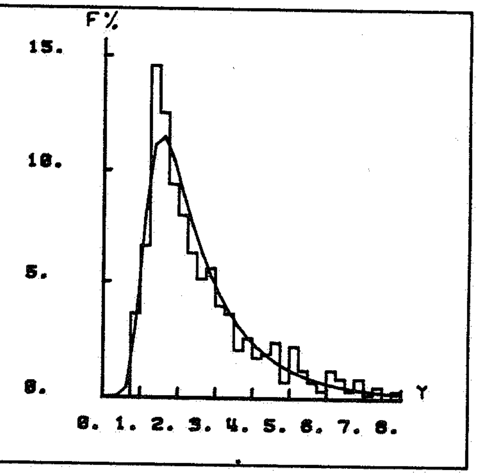
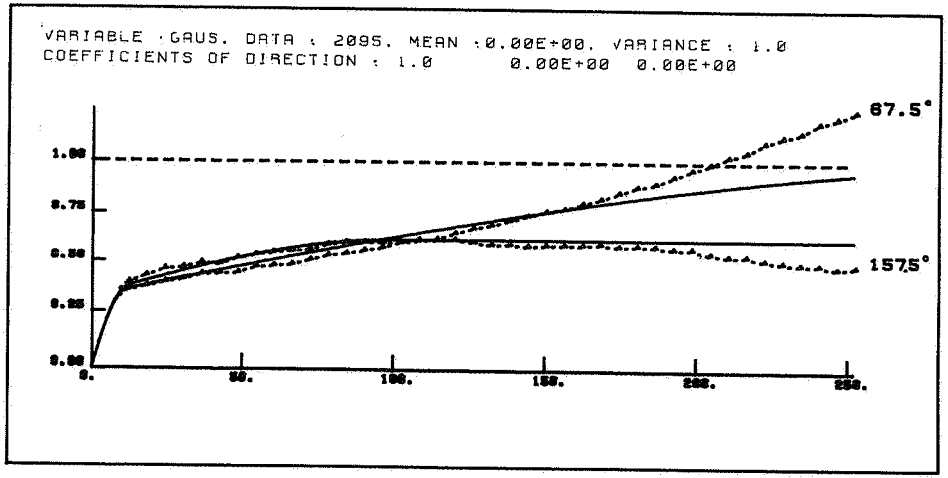


Figure 2b. Experimental histogram of blocks (583) and model of permanence $C_n(v) = C_n . r^n$



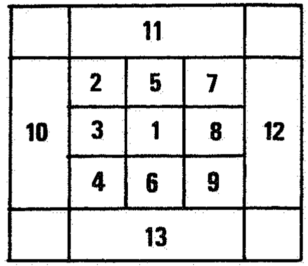
NUGGET: 0.00E+00	NB. STRUCTURE(S): 3		ANISOTROPY	
SPH C: 0.32	A: 12.	X: 1.0	Y: 1.0	Z: 0.00E+00
SPH C: 0.29	A: 0.30E+03	X: 1.0	Y: 3.0	Z: 0.00E+00
SPH C: 0.39	A: 0.35E+03	X: 1.0	Y: 0.00E+00	Z: 0.00E+00

Figure 3. Variograms of blast-hole data and fitted models.

VARIOGRAMS

After an extensive study of the variograms, we chose to fit a model on the anamorphosed values of the 2 095 blast-holes. Figure 3 shows the model adopted for anamorphosed values. In the 2nd and 3rd structures we observed a considerable geometric and zonal anisotropy between the two main directions: 67.5 NNE and 157.5 WNW. We note that the 3rd structure will not be used, and that a non negligible drift can be observed on a large scale.

The chosen kriging neighborhood for 5 x 5 panels involves 13 weighting factors, where each outside weighting factor regroups 3 panels.



TESTING THE FORMULA: $E[H_n(X)/Y]=\rho^n H_n(Y)$

This formula has to be verified when the couple of two standardized Gaussian variables X and Y is a Gaussian bivariate with correlation coefficient ρ.

This formula is of paramount importance in the theory of non-linear geostatistics and one is very often led to postulate the binormality of couple of Gaussian variables for the sole purpose of using this formula. This test is a useful mean of judging the suitability of the bigaussian hypothesis for non-linear geostatistics.

In the present study, it has been possible to apply this test to the couples of point Gaussian variables Y(x) and Y(x+h). The 2 095 data were anamorphosed and Figure 4 shows the results for h = 10 m ± 1 m and n up to 4. On the abscissa we put $-2 < Y < 2$ (which is more than 95% of a Gaussian distribution). The left-hand side of the formula is represented by a continuous line, and the dashed line represented what was expected, i.e. $\rho^n H_n(Y)$.

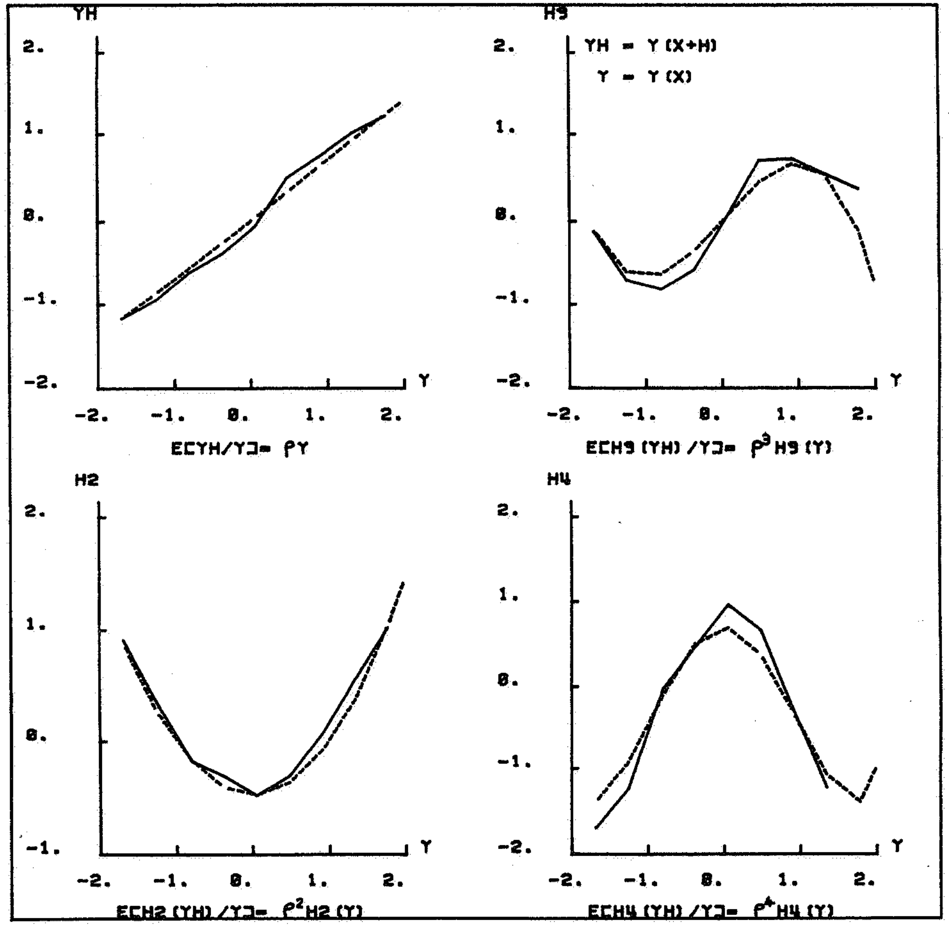


Figure 4. Test of $E[H_n(Y(x+h)|Y(x)] = \rho^n H_n[Y(x)]$.

ESTIMATION OF RECOVERABLE RESERVES

The three methods were used to estimate the proportion P of ore and the metal quantity Q that would be recovered from a set of 30 x 30 m panels with a certain cut-off grade on 10 x 10 selection units. The value of P will be expressed in % while Q will be given using the tonnage of one panel as the unit. Note that when the cut-off grade is zero, Q represents the sum of the quantities of metal in each panel.

Experience has shown that the multigaussian method and disjunctive kriging both give results which are satisfactory globally, i.e. over the whole group of panels in the level where the structural analysis and the anamorphosis are defined. We can reasonably expect U.C. to give equally satisfactory results.

In our study, the cut-off grade has only been applied to those blocks in the well sampled regions, but this group is not representative of the whole of the level. For example the mean of the 50 panels is 2.45% whereas that of the level is 2.30%. Comparisons between the three methods and also with the actual figures were made for four different sets of panels:

- the 50 panels in the well-sampled regions
- the 12 panels in a low grade zone in the north (where the grade of each panel < 1.5%)
- the 6 panels in a rich zone (where the grade ⩾ 3.2%)
- the 15 panels in a mixed zone (8 panels have grades between 1.5 and 3.2% while the other 7 are high grade panels).

The results are presented on Figures 5, 6, 7 and 8. Comparisons between the three methods show that the estimates are very similar, except for several cases in the last group of panels. This suggests that uniform conditioning gives global results which are just as good as the other two methods, as was indicated earlier.

Moreover the estimated recovery curves are quite close to those obtained from the real values. However it should be noted that the quantity of metal recovered is underestimated when the set of panels contains rich blocks. In particular, when the cut-off grade is zero, the actual grade for the various zones is generally higher than the estimated one. See Table 2 below.

The average grade of the rich panels is more underestimated than in that its value is closer to the average of all the panels (2.30%). This effect is probably due to the strong assumptions about the stationarity made by all the methods, even though this effect is not evident for the poor panels.

TABLE 2. Comparison of actual grades with estimates for a cut-off grade of zero.

	REAL GRADE	ESTIMATED GRADE
All 50 panels	2.45	2.34
The 12 poor panels	1.16	1.12
The 6 rich panels	4.65	4.20
The 15 mixed panels	2.87	2.91

Experience has shown that kriging with a universality condition underestimates the actual grade of rich panels (because of its smoothing influence). But we also know that since the

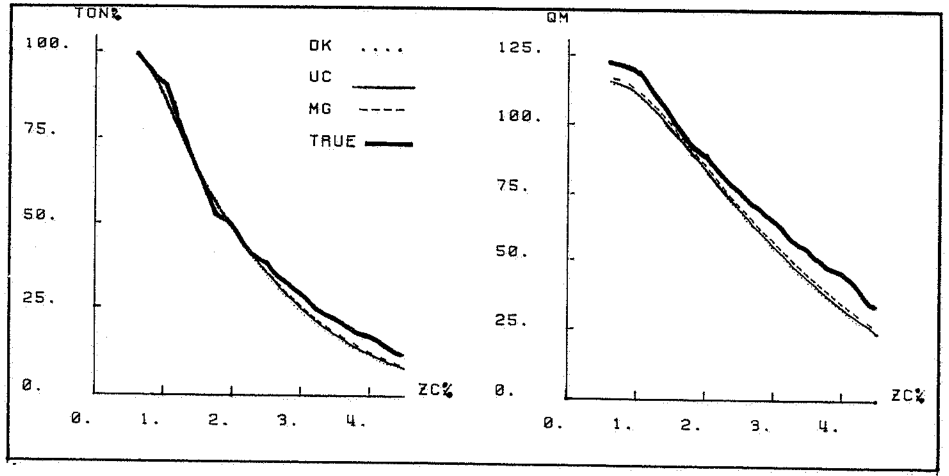


Figure 5. Tonage recovered / Cut-off grade and Recoverable quantity of metal / Cut-off grade for 50 panels.

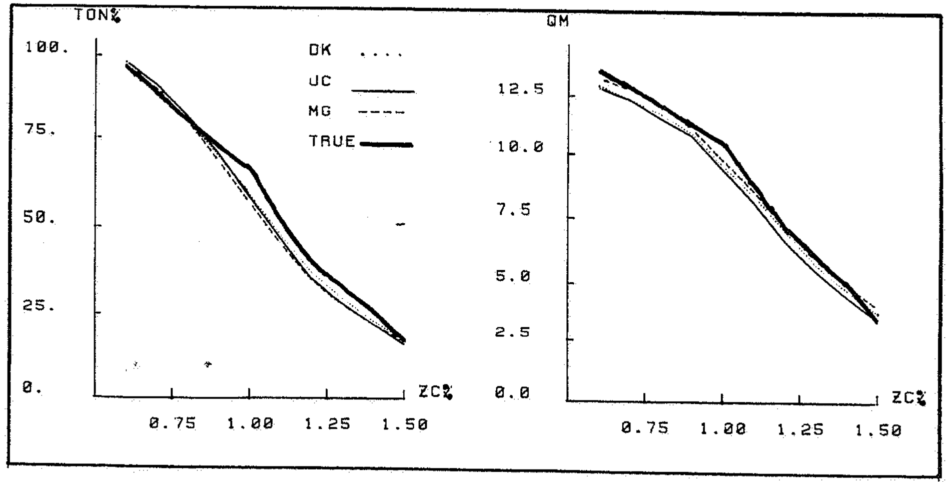


Figure 6. Same as Fig. 5, for 12 panels in a poor area.

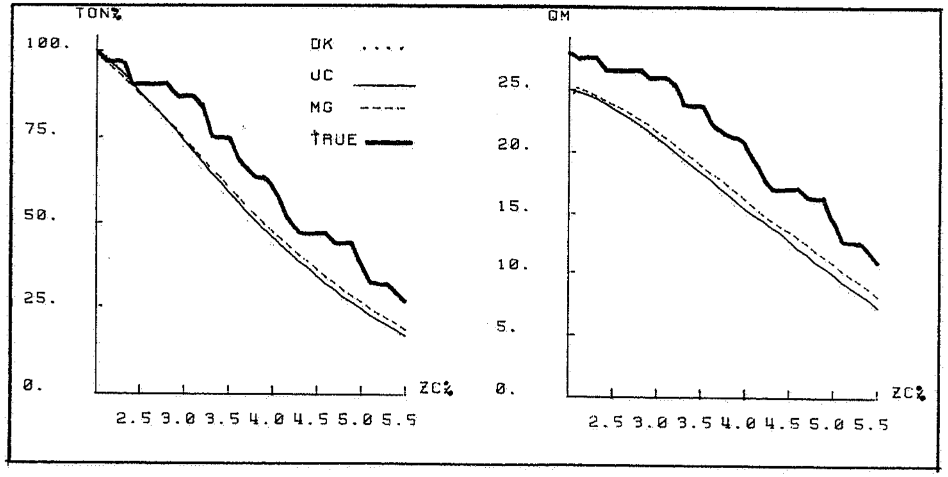


Figure 7. Same as Fig. 5, for 6 panels in a rich area.

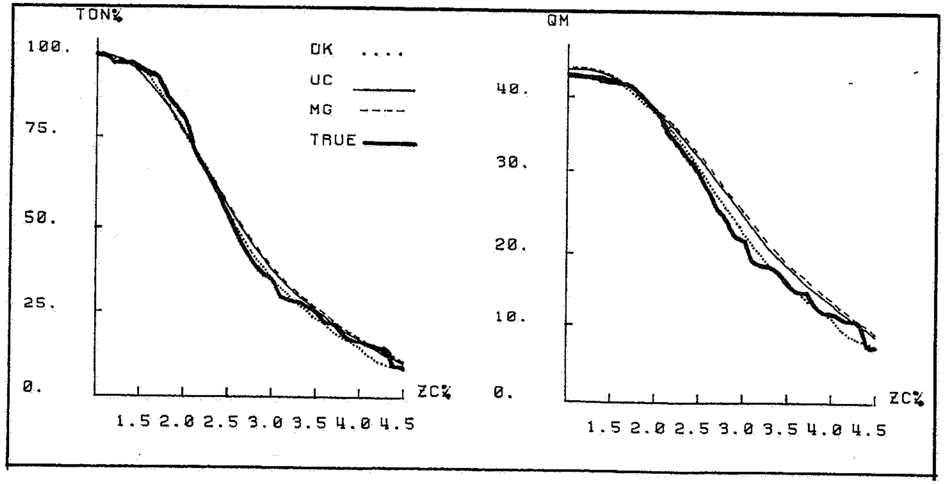


Figure 8. Same as Fig. 5, for 15 panels in a mixed area.

objective in using kriging is to avoid systematic errors when making decisions about the grades, this is of no importance in practice. What is important is that the panels with high estimated grades should be neither over- or under- estimated on average rather than that the truly rich panels be underestimated.

It is well known in ordinary kriging that if a selection is being made on the actual grades of panels (rich ones for example) the average grades and the recoverable reserves are underestimated. The same observation is true in the non-linear case.

This is why, in order to compare estimates and reality, we decided to work on a geographical zone, and not on a population of panels chosen because of their real grades.

Nevertheless, the relatively high discrepancies noticed in the rich area suggest that there is a need for adapting the estimators locally, taking into account local departure from stationarity (such as local trends). Although some preliminary work has been done in that field, it is still open for investigation.

CONCLUSION

This study has made it possible to compare three different non-linear methods both from the theoretical and practical points of view. It has shown that they are very similar.

The uniform conditioning technique first proposed by Matheron [2] in 1975 gave quite satisfactory results and therefore merits further study, since it is easier to use than the other methods. For example it avoids the problems of inacceptable values of the probability density function at high or low grades that sometimes occur with disjunctive kriging. As the results obtained from both methods are very similar, the relative simplicity of uniform conditioning and its greater efficiency in terms of computing requirements come to the fore. The greatest advantage of uniform conditioning is undoubtedly that only one kriging is required to obtain the same results and also that it does not require any additional hypotheses: all that is required is the joint normality of the couples (Y_v, Y_α) and thus of the couples $(Y_v, \sum_\alpha \lambda_\alpha Y_\alpha)$; the "permanence" formulae, i.e. the anamorphosis ϕ_r, is used only at the level of the blocks v, and not of the panels [3, p. 20].

On the negative side, since it depends on only one kriging, two panels with the same kriged estimate and the same kriging neighborhood would then be attributed the same distribution of block grades, whereas one would expect this only if the sample grades were the same.

It is also important to note that although these three methods are fairly sophisticated from the theoretical point of view, they give consistent results and can be used in routine studies.

A current research project, funded by a grant from the French Government concerns the stationarity hypotheses required, with a view to relaxing them whenever possible. The aim of this particular study was to highlight the similarities between the three methods which, up till now, have always been regarded as being quite dissimilar. In fact there is substantial core common to all three, and so rather than trying to find ways of relaxing the stationarity requirements for each of these methods separately, it is better to study them all together.

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