Foliations fields and 3D cartography in geology : Principles of a method based on potential interpolation.

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Abstract

A modeling method which takes into account known points on a geological interface and plane orientations data such as stratifications or foliations planes is described and tested.The orientations data do not necessarly belong to one of the interfaces but are assumed to sample the main anisotropy of a geological formation as in current geological situations.The problem is to find the surfaces which pass through the known points on interfaces and which are compatible with the orientations data. The method is based on the interpolation of a scalar field defined in the space the gradient of which is orthogonal to the orientations, given that some points have the same but unknown scalar value (points of the same interface), and that scalar gradient is known on other points (foliations). The modeled interfaces are represented as isovalues of the interpolated field. Preliminary two-dimensional tests carried-out with different covariance models demonstrate the validity of the method , which is easilly transposable in three dimensions.

Keywords : 3d-Modeling, potential,Vector field, Interpolation

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Introduction

Describing the 3-dimensional geometry of geological objects is a preliminary for understanding and simulating geological processes. A geological model can be considered as a set of formations (volumes) bounded by interfaces (surfaces) and cut by faults (surfaces). Faults do not necessarily bind formations.

The general goal of 3D cartography is to produce models which are (1) geometrically correct i.e. fitting known geometrical features,(2) topologically consistent i.e. respecting the relations between different components of a geological object, and (3) geologically realistic.

Due to the diversity of data and the complexity of geological bodies, specific methods have to be developed for each kind of situation. Classical geostatistical methods ([3]) have proved their efficiency when enough data points are known on a relatively simple interface. Interpolation methods such as DSI ([9]) , Bezier Surfaces ([1]) have also been applied to geometric surface modeling.

In this paper we propose a method which can be used when some points are known on one or more interfaces, and when additional plane orientation data are available. Note that these orientation data do not necessarily belong to one of the interfaces but are assumed to sample the main anisotropy of a geological formation (sedimentary plane, foliation, cleavage plane). This is one of the most common situations in 3D mapping. The problem is to find surfaces which pass through the known points of each interface and which are compatible with the orientation data.

1 Methodology

1.1 General conditions

(1) **Firstly we** assume that the interfaces to be modeled belong to a family of parallel surfaces nearly following the foliation field. This assumption is reasonable in many cases:

- In folded sedimentary rocks it is reasonable to assume that there is a general parallelism between the stratification and the lithologic interfaces.
- The shape of plutonic borders are very similar to the internal structures of the field. Though borders and structures are not always strictly parallel as in syntectonic plutons ([4]), internal structures are widely used to determine the plutons shape (see [5] and [7]).
- In metamorphic series the main lithologic interfaces display a strong parallelism with metamorphic foliations.

(2) **Secondly we** assume that some of the orientation data can be transformed into vectorial data. In geological terms this means that polarity of structures must be known in some places.

1.2 Review

Some attemps have been made based on drawing trajectories ([2] and [8]). These rely on an integration method. Starting from one point the method consists of iteratively recomputing the position of the following point by interpolating the vector field at the current position. This method gives an idea of what the geometry looks like but cannot be directly used to model the geometry of an interface itself since it is very sensitive to small errors which cumulate when moving off the starting point. Thus two distinct starting points on the same interface will lead to two distinct trajectories. The method can be improved by correcting the trajectory when more than one point is known ([6]) but still remains unrobust since controlling the drift of trajectories is very difficult. Furthermore it is not simple to extend the method to 3D.

1.3 Basic principle

The method described in this paper is based on the following idea : a scalar field is defined in the space, whose gradient is orthogonal to the orientation data. The idea is to interpolate the scalar field given that some points have the same but unknown scalar value (points of the same interface), and that either the gradient of the scalar field or a direction orthogonal to this gradient is known at other points. Finally the modeled interfaces are represented as isovalues of the interpolated field.

For preliminary tests of validity (feasibility) the method is described and tested in 2 dimensions. Isovalues are therefore displayed as curves.

2 Methodological options.

2.1 Position of the problem.

In the rest of the paper, $\mathbf{x}$ will be a generic point in the two-dimensional space $\mathbb{R}^2$. Small characters will be used when the coordinates will be needed. For instance $\mathbf{x} = (x, y)$. The objective is to generate a family of smooth curves (surfaces in 3-D) compatible with the data. Generally these will be given in an implicit form:

$$C_\alpha = \{\mathbf{x} : \psi_\alpha(\mathbf{x}) = 0\}$$

The data consist of:

Orientation data: At point $\mathbf{x}_i$, there should be only one curve passing through $\mathbf{x}_i$, the curve must be differentiable at that point, and one tangent to it must be parallel to one assigned direction τ_i.

Interface data: There are sets of points J_k, all of which must be assigned to the same curve:

$$\forall k \ \exists \alpha \text{ (depending on } k) \text{ such that } \ \forall j, j' \in J_k \quad \psi_\alpha(\mathbf{x}_j) = \psi_\alpha(\mathbf{x}_{j'}) = 0$$

Here k is an index to the interface.

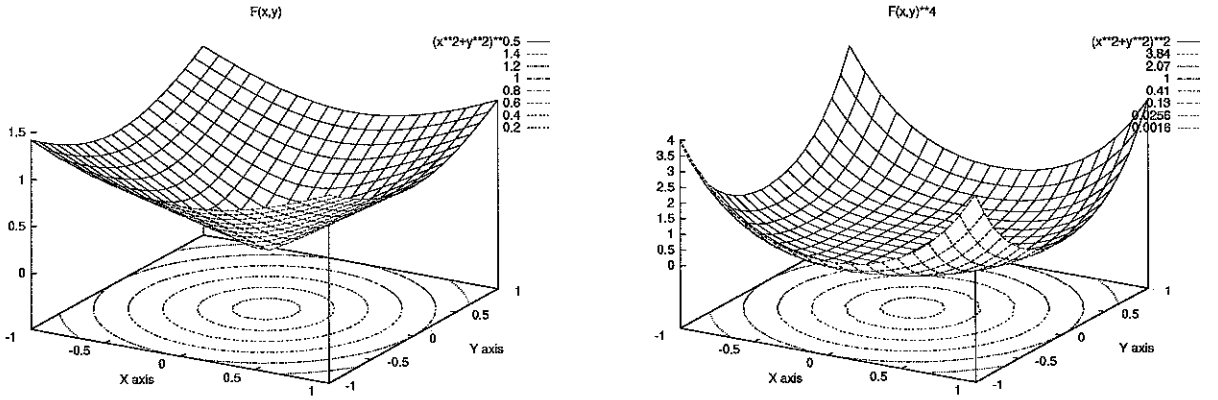

Figure 1: *The curve family generated by the isolines of the two above functions are the same.*

2.2 Simplifications to the problem.

Our method assumes that the curve family can be described in the following more restrictive form:

$$C_\alpha = \{\mathbf{x} : \psi(\mathbf{x}) = \alpha\}$$

where ψ is differentiable. Note that this implies that the curves are themselves differentiable, and that they do not intersect.

It is clear that ψ will play the role of a potential in the methodology. There is a large degree of freedom in its choice. For instance any monotonic transformation f can give rise to another parameterization:

$$\phi = f \circ \psi$$

that is strictly equivalent from our point of view, in the sense that it will generate the same collection of curves, and hence the same solution. Figure 1 shows this for the function $\psi(\mathbf{x}) = |\mathbf{x}|$ and $\phi = |\mathbf{x}|^4$.

The function ψ will therefore be termed conventional potential in the rest of the paper. We emphasize that this does not mean that a potential exists in the geological sense.

Within this setting, the gradient of ϕ can be calculated at any point and it gives the direction normal to the iso-curves. Note that the gradient carries much more information than do the tangents, because its sign determines the outer direction (hence the polarization). Moreover its modulus tells us whether the isocurves tend to become closer or more distant to one another locally. In practice the outer direction is generally known, and the compression of the layers can be measured. Hence the gradient is approximately known. So we shall consider that gradient data are known at some places and that only the tangents are at some other data points.

2.3 The geostatistical method.

The conventional potential will be assumed to be a realization of a differentiable random function, that will be written Z to make the distinction clear. Covariance models for

it will be considered later in the paper. Kriging will be used to produce estimates of Z on the whole domain. The reader is assumed to be familiar with kriging in the multivariate case when there is a drift component. See [10] for a monograph on these topics. See also [11] for the theory of the IRF-k model which is used at some places in the paper. The cokriging that we are using is not a standard one, because the variables we are dealing with are algebraically linked. This calls for specific treatment. We are going to give some details on this in this paragraph.

The data to be used in the kriging consist of:

Gradient data :

$$\frac{\partial Z}{\partial x}(\mathbf{x}_i) = G_i^x$$
$$\frac{\partial Z}{\partial y}(\mathbf{x}_i) = G_i^y$$

Data on tangents . The tangent vector will be considered to be a fixed direction, and the scalar product to be a zero data :

$$< \nabla Z(\mathbf{x}_{i'}) , \tau_{i'} > = 0$$

The increments : for each set J_k

$$Z(\mathbf{x}_j) - Z(\mathbf{x}_{j'}) = 0 \qquad \forall (j, j') \in J_k$$

These will be used as data for each independent pair (j, j'). There are $\mathrm{Card}(J_k)-1$ of them for each discretized curve k. For instance one can choose the first point j in J_k arbitrarily and consider every pair (j, j') for $j' \neq j$ in J_k.

Note that, while the contribution of the increment data seems to vanish on the kriging result, because obviously if $\lambda_{jj'}$ is the weight associated to the increment defined by the points $\mathbf{x}_j$ and $\mathbf{x}_{j]}$ we have :

$$\lambda_{jj'} \left(Z(\mathbf{x}_j) - Z(\mathbf{x}_{j'})\right) = 0$$

the information they carry actually contributes to the final estimate, as they are taken into account by the kriging equations. Indeed, due to the fact that kriging is an exact interpolator, one has:

$$Z^K(\mathbf{x}_j) - Z^K(\mathbf{x}_{j'}) = 0 \qquad \forall (j, j') \in J_k$$

so that the same isopotential curve will pass through all points in the set J_k. The same holds for the tangent equations as well.

2.4 The kriging solution.

Because the data consist of first order increments of Z, the conventional potential is only defined up to an arbitrary constant. Thus we shall choose an arbitrary origin $\mathbf{x}_0$, and shall estimate only the increments $Z(\mathbf{x}) - Z(\mathbf{x}_0)$:

$$[Z(\mathbf{x}) - Z(\mathbf{x}_0)]^K = \sum_{i \in I} (\lambda_i G_i^x + \mu_i G_i^y) + \sum_{i' \in I'} \nu_{i'} < \nabla Z(\mathbf{x}_{i'}), \tau_{i'} > + \sum_k \sum_{j,j' \in \mathcal{P}(J_k)} \lambda_{jj'} [Z(\mathbf{x}_j) - Z(\mathbf{x}_{j'})]$$

(it is convenient to write the contribution of $Z(\mathbf{x}_j) - Z(\mathbf{x}_{j'})$ explicitly to derive the kriging equations, even if this contribution is 0 in the very end). In this equation, $\mathcal{P}(J_k)$ is one set of independent pairs associated with J_k.

We shall need the covariances and cross-covariances of each function involved. These can be obtained from the covariance of Z, that we shall denote K_Z in the rest of the paper. Let $\mathbf{h}$=$\mathbf{x}$-$\mathbf{y}$ be the vector joining the two points $\mathbf{y}$ and $\mathbf{x}$, $r = |\mathbf{h}|$ its modulus, $h_x =< \mathbf{x}\text{-}\mathbf{y}, \mathbf{e}_x >$ its component along x, and h_y along y. If K_Z is isotropic, (this assumption is not necessary to the method but simplifies the formulae), then we can write :

$$K_Z(\mathbf{h}) = C_Z(r)$$

In order for Z to be differentiable, K_Z must be twice differentiable. Now if C_Z is twice differentiable for $r \geq 0$ then K_Z is itself twice differentiable if $C_Z'(0) = 0$. Under these conditions, the required covariances are given by:

$$\begin{aligned}
K_{Z\,G^x}(\mathbf{x}\text{-}\mathbf{y}) &= \mathrm{Cov}\left(Z(\mathbf{x}), Z_x'(\mathbf{y})\right) = -\frac{h_x}{r} C_Z'(r) \\
K_{G^x\,G^y}(\mathbf{x}\text{-}\mathbf{y}) &= \mathrm{Cov}\left(Z_x'(\mathbf{x}), Z_y'(\mathbf{y})\right) = \frac{h_x\,h_y}{r^2}\left(\frac{1}{r}C_Z'(r) - C_Z''(r)\right) \\
K_{G^x}(\mathbf{x}\text{-}\mathbf{y}) &= \mathrm{Cov}\left(Z_x'(\mathbf{x}), Z_x'(\mathbf{y})\right) \\
&= \left(\frac{{h_x}^2}{r^3} - \frac{1}{r}\right) C_Z'(r) - \left(\frac{h_x}{r}\right)^2 C_Z''(r)
\end{aligned}$$

Given these expressions there is no difficulty in writing the kriging system. The universality conditions still require some care because we must take account of the fact that G is the gradient of Z. The estimation error of the increment $Z(\mathbf{x}) - Z(\mathbf{x}_0)$ must be authorized. This means that the kriging weights have to be such that, for any drift function f_l we have:

$$\begin{aligned}
f_l(\mathbf{x}) - f_l(\mathbf{x}_0) &= \sum_{i\in I}(\lambda_i \frac{\partial}{\partial x} f_l(\mathbf{x}_i) + \mu_i \frac{\partial}{\partial y} f_l(\mathbf{x}_i)) + \sum_{i'} \nu_{i'} < \nabla f_l(\mathbf{x}_{i'}), \tau_{i'} > \\
&\quad + \sum_k \sum_{jj'\in\mathcal{P}(J_k)} \lambda_{jj'}(f_l(\mathbf{x}_j) - f_l(\mathbf{x}_{j'}))
\end{aligned}$$

If the drift space is the set of polynomials of degree k, the above equation must hold for each of the monomials f_l which span the drift space. Note that for a constant drift $f_0 = 1$, the condition is automatically fulfilled, so there is one universality condition which vanishes from the kriging system.

Note also that as the estimation error is authorized, all the calculations remain valid if Z is a differentiable IRF-k and C is its generalized covariance.

For efficiency reasons, and because we are not interested in the kriging variance in this application, we prefer to use the dual form of kriging. Then the interpolator has the form:

$$\begin{aligned}
Z(\mathbf{x})^K &= A_0 + \textstyle\sum_{i\in I} (a_i K_{Z\,G^x}(\mathbf{x}\text{-}\mathbf{x}_i) + b_i K_{Z\,G^y}(\mathbf{x}\text{-}\mathbf{x}_i)) \\
&\quad + \textstyle\sum_{i'\in I'} c_{i'} (\tau_{i'}^x K_{Z\,G^x}(\mathbf{x}\text{-}\mathbf{x}_{i'}) + \tau_{i'}^y K_{Z\,G^y}(\mathbf{x}\text{-}\mathbf{x}_{i'})) \\
&\quad + \textstyle\sum_k \sum_{jj'\in\mathcal{P}(J_k)} d_{jj'}(K_Z(\mathbf{x}\text{-}\mathbf{x}_j) - K_Z(\mathbf{x}\text{-}\mathbf{x}_{j'})) \\
&\quad + \textstyle\sum_{l>0} e_l f_l(\mathbf{x}) \qquad (1)
\end{aligned}$$

where A_0 is a constant that disappears in the final result. So it can be taken to be 0. The x component of the estimated gradient is given by:

$$\begin{aligned}\left(\frac{\partial}{\partial x}Z(\mathbf{x})\right)^K &= \sum_{i\in I}\left(a_i K_{G^x}(\mathbf{x}\text{-}\mathbf{x}_i) + b_i K_{G^x G^y}(\mathbf{x}\text{-}\mathbf{x}_i)\right)\\ &+ \sum_{i'\in I'} c_{i'}\left(\tau^x_{i'} K_{G^x}(\mathbf{x}\text{-}\mathbf{x}_{i'}) + \tau^y_{i'} K_{G^x G^y}(\mathbf{x}\text{-}\mathbf{x}_{i'})\right)\\ &+ \sum_k \sum_{jj'\in\mathcal{P}(J_k)} c_{jj'}(K_{G^x Z}(\mathbf{x}\text{-}\mathbf{x}_j) - K_{G^x Z}(\mathbf{x}\text{-}\mathbf{x}_{j'}))\\ &+ \sum_{l>0} d_l \frac{\partial}{\partial x} f_l(\mathbf{x})\end{aligned}$$

The kriging equations are obtained by writing that the interpolator is exact at points where the gradient is known, has zero scalar products where the tangents are known, and has zero increments along the sets J_k. The universality conditions give additional equations which complete the kriging system. The detailed equations are given in the appendix for the case where only gradient and interface data exist.

The isocurves of Z^K satisfy the constraints:

$$\begin{cases} \bigtriangledown Z^K(\mathbf{x}_i) = G_i & \forall i \in I \\ < \bigtriangledown Z^K(\mathbf{x}_{i'}), \tau_{i'} > = 0 & \forall i' \in I' \\ Z^K(\mathbf{x}_j) - Z^K(\mathbf{x}_{j'}) = 0 & \forall j, j' \in J_k \end{cases}$$

as required.

2.5 Models for the conventional potential.

Convolution by a differentiable ordinary function is a very general way of getting differentiable models. This however does not generally give closed form formulae that can be used easily in a program. Here we mention one stationary model and an IRF-2 one on which tests have been conducted.

The first one is the well known gaussian model. It has a spatial scale parameter (a) and is infinitely differentiable. If $C(r) = \exp\{-(r/a)^2\}$, we find :

$$\begin{aligned} K_{Z G^x}(\mathbf{h}) &= -2\frac{h_x}{a^2} C(r) \\ K_{G^x}(\mathbf{h}) &= \left(\frac{2}{a^2} - 4\frac{x^2}{a^4}\right) C(r) \\ K_{G^x G^y}(\mathbf{h}) &= -4\frac{h_x h_y}{a^4} C(r) \end{aligned}$$

It is well known that due to its infinite differentiability this model has unstable behavior. It is safest to introduce a little nugget effect in the data covariances, which is anyway more realistic since the data have limited precision.

The second one was obtained by Wendum [12] while defining a non divergent spline interpolator in $I\!R^2$. It has the form:

$$C_Z(r) \;=\; r^4 \,\mathrm{Log}(r)$$

and is the generalized covariance of a differentiable IRF-2 (in $I\!R^2$). This model does not have a scale parameter. If λ and μ are two authorized measures of degree 2, and $r = |\mathbf{x}\text{-}\mathbf{y}|$, we obviously have :

$$\int \lambda(dx)\,\mu(dy)\,r^4 \;=\; 0$$

If $C_a(r) = (r/a)^4 \mathrm{Log}(r/a)$ this implies :

$$\int \lambda(dx)\,\mu(dy) C_a(r) \;=\; \frac{1}{a^4} \int \lambda(dx)\,\mu(dy)\,C_1(r)$$

Hence, the kriging weights would be the same if we were using C_a instead of C_1 and so there is no point in introducing a scale parameter in this model.

The derivation of the kriging solution makes use of the following expressions :

$$\begin{aligned} K_{Z\,G^x}(\mathbf{h}) &= -h_x\, r^2\,(1 + 4\,\mathrm{Log}(r)) \\ K_{G^x}(\mathbf{h}) &= -6\,{h_x}^2 - r^2 - (4r^2 + 8{h_x}^2)\,\mathrm{Log}(r) \\ K_{G^x\,G^y}(\mathbf{h}) &= -2h_x h_y\,(3 + 4\,\mathrm{Log}(r)) \end{aligned}$$

3 Application

The data set that will be used in this section consists of series of vectors which sample a syncline and anticline structure. It consists of 8 gradient data, for which the modulus had been fixed to unity, which correspond to a cylindrical 3-D structure, and 2 interfaces sampled respectively at 3 and 2 points.

The method is tested under various conditions. The influence of the covariance model will be shown first, then the sensitivity to the gradient modulus will be illustrated. Finally an example of the results when tangent data are used in addition to the gradient will be given.

3.1 Influence of the covariance model.

The estimation of a covariance model for this method is not the suject of this paper. Instead of that we shall only illustrate the impact of the model choice on the results.

The gaussian model has a range parameter and the drift can be freely chosen. The results of modifying these parameters are shown in Figure 2. The first line on this figure corresponds to a silly choice, for the range is less than the data spacing. As a result the interpolator is almost equal to the drift term except near the gradient data. The difference between the second and the third columns (drift of degree 1 and 2) is very small, because a linear drift is appropriate in this case. A range $a = 5$ and a drift of degree 1 or 2 seem to be appropriate choices on this example.

The spline model gave the results on the left hand side of figure 3. They are very similar to the results obtained with the gaussian model when appropriate choices are made concerning the range.

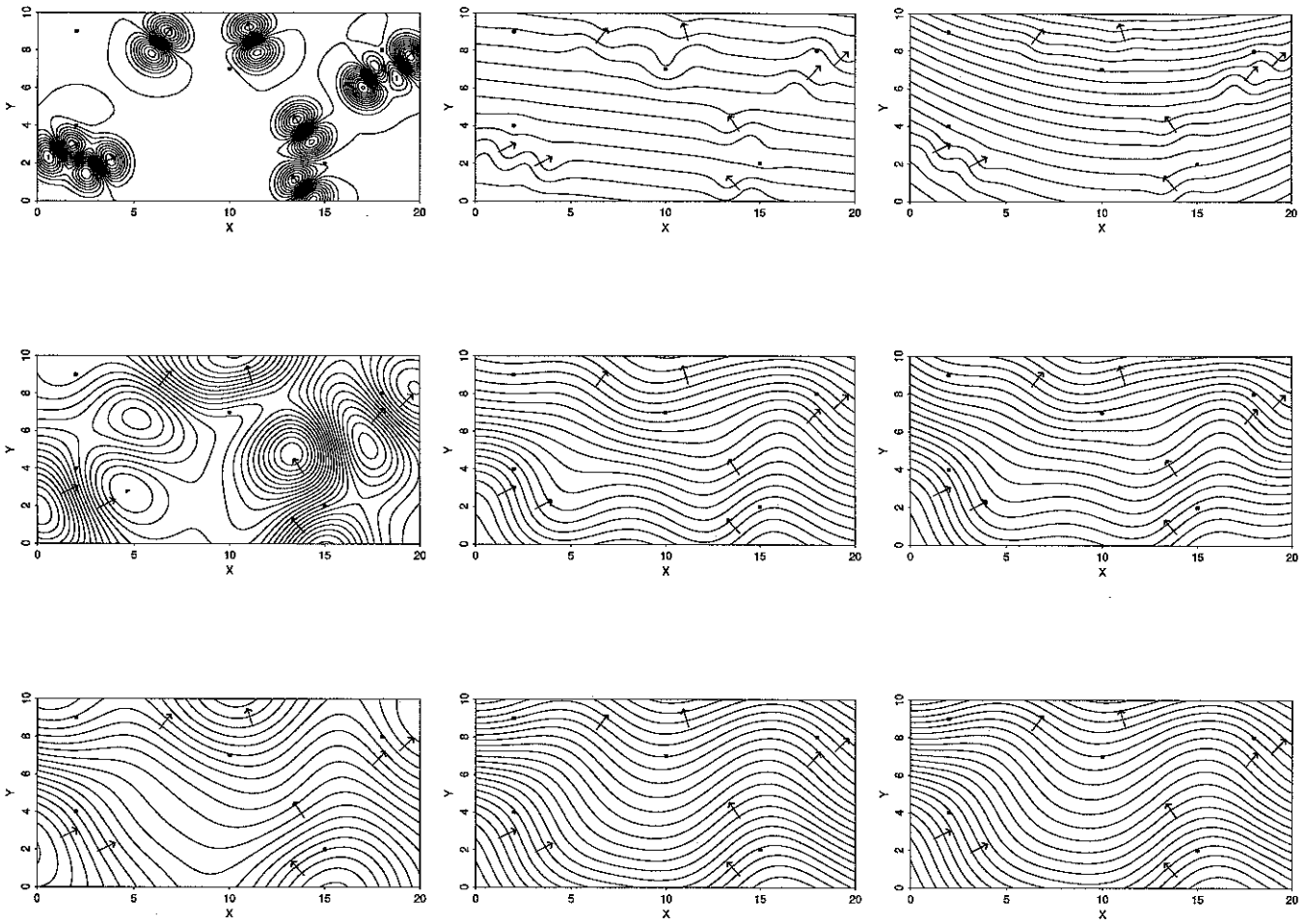

Figure 2: *Field generated by a gaussian covariance with different ranges, under drift of degree 0, 1 and 2. The dot shows the interface data, and the arrows the gradient data. The first line correspond to a unit range, the second one to a range $a = 3$ and the third to $a = 5$. The first column is with constant drift, the second one with a planar drift, and the third one with a second degree drift.*

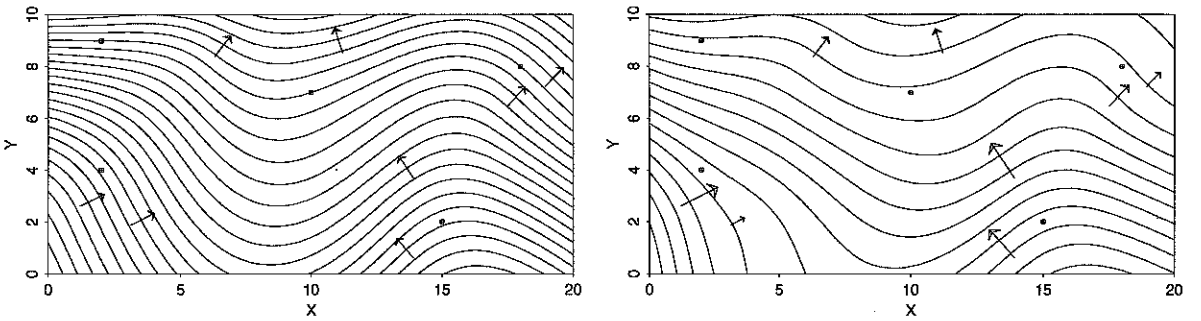

Figure 3: *Results from random perturbation of the modulus of the gradient data. Original interpolation on the left and perturbed one on the right.*

3.2 Influence of the modulus of the gradients.

The moduli of the gradient data are to some extent arbitrary. In the example from the previous section, they were fixed to unity. In this section, we show the results from a random generation with uniform distribution in the interval $[0.4, 1.6]$, and independence from one data to another. Figure 3 shows the results if the interpolation is based on the 8 gradient data used so far. Figure 4 shows the effect of this randomization when 37 gradient data are used.

The modulus has an obvious effect on the spacing of the isolines, without disturbing the overall picture.

3.3 Using tangent data.

Figure 5 shows the results obtained when the original 8 gradient data are joined to 29 tangent constraints corresponding to the additional points of the previous section. The results are to be compared with the left part of Figures 3 and 4. The data on the tangents is effectively used, but it is obviously less informative than the gradient. Would it be possible to use only data on tangents instead of gradient data ? The answer is of course negative, for it would amount to do a universal kriging with only zero data. The kriging would find the trivial constant solution in this case.

4 Conclusion and future work.

The tests carried out in the paper demonstrate the feasibility of the method. It gives realistic results compared to what can be expected from the geological data. This method has the advantage of combining in a same interpolator different types of independent geological information : known points on interfaces, orientation data at other points and also gradient data. Another interesting feature is its ability to model a family of surfaces, simultaneously taking into account all known points on different interfaces. Furthermore it can be applied in 3D without modifying its principle.

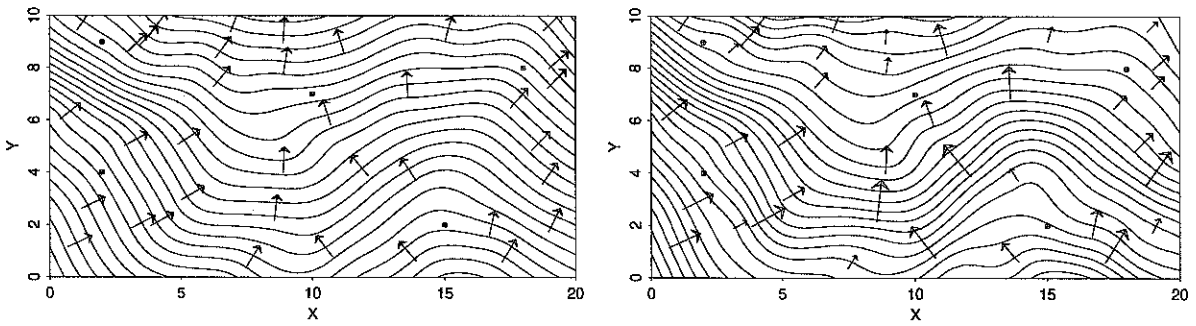

Figure 4: *Same as the previous figure, when the original gradient data consist of* 37 *vectors.*

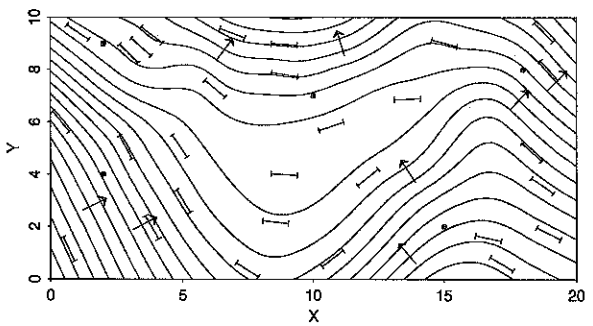

Figure 5: *Using the spline model, with gradient and tangent data.*

Probably the major drawback is linked with the regularity of the model. This means that singularities such as angles (non differentiability) faults (discontinuities) need special modifications. The second point which deserves additional work is the model estimation in this context.

Finally the method appears to be promising and will be really efficient for geological modeling in 3D.

5 Acknowledgments

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APPENDIX- The kriging equations

We shall derive the kriging equations for the dual form for the case where the directional data consist of gradient data only. The generalization is straightforward and results in less readable formulae. Starting from the expression given by the equation 1, we specify that the interpolator is

- Exact at the gradient data points:

$$\begin{aligned}\forall i \quad & \sum_{i' \in I} (a_{i'} K_{G^x}(\mathbf{x}_i - \mathbf{x}_{i'}) + b_{i'} K_{G^x G^y}(\mathbf{x}_i - \mathbf{x}_{i'})) \quad + \\ & \sum_k \sum_{jj' \in \mathcal{P}(J_k)} c_{jj'} (K_{G^x Z}(\mathbf{x}_i - \mathbf{x}_j) - K_{G^x Z}(\mathbf{x}_i - \mathbf{x}_{j'})) \quad + \\ & \sum_{l>0} d_l \frac{\partial}{\partial x} f_l(\mathbf{x}_i) \quad = \quad G_i^x\end{aligned}$$

  (We do not write the equation for the second component of the gradient, because it is very similar.)

- Gives zero increments at $j^{"}, j^{'''} \in \mathcal{P}(J_{k'})$, that is:

$$\begin{aligned}\sum_{i \in I} & \{a_i (K_{Z G^x}(\mathbf{x}_{j^{"}} - \mathbf{x}_i) - K_{Z G^x}(\mathbf{x}_{j^{'''}} - \mathbf{x}_i)) \quad + \\ & b_i (K_{Z G^y}(\mathbf{x}_{j^{"}} - \mathbf{x}_i)) - (K_{Z G^y}(\mathbf{x}_{j^{'''}} - \mathbf{x}_i))\} \quad + \\ \sum_k \sum_{jj' \in \mathcal{P}(J_k)} & c_{jj'} \{(K_Z(\mathbf{x}_{j^{"}} - \mathbf{x}_j) - K_Z(\mathbf{x}_{j^{'''}} - \mathbf{x}_{j'}) - \\ & K_Z(\mathbf{x}_{j^{'''}} - \mathbf{x}_j) + K_Z(\mathbf{x}_{j^{'''}} - \mathbf{x}_{j'}))\} \quad + \\ & \sum_{l>0} d_l (f_l(\mathbf{x}_{j^{"}}) - f_l(\mathbf{x}_{j^{'''}})) \quad = \quad 0\end{aligned}$$

- And satisfies the equations that come from the universality conditions:

$$\begin{aligned}\sum_{i \in I} & a_i \frac{\partial}{\partial x} f_l(\mathbf{x}_i) + b_i \frac{\partial}{\partial y} f_l(\mathbf{x}_i) \\ + \sum_k \sum_{jj' \in \mathcal{P}(J_k)} & c_{jj'} (f_l(\mathbf{x}_j) - f_l(\mathbf{x}_{j'})) \quad = \quad 0\end{aligned}$$

This has to be written for $l = 1, ..N$.

This can be written in a more synthetic way, if we agree to let $K_{G^x}^{i'i}$ denote $K_{G^x}(\mathbf{x}_i - \mathbf{x}_{i'})$. First we define the covariance matrix:

$$K = \left(\begin{array}{c|c|c} K_{G^x}^{i'i} & K_{G^xG^y}^{i'i} & K_{G^xZ}^{ji} - K_{G^xZ}^{j'i} \\ \hline K_{G^yG^x}^{i'i} & K_{G^y}^{i'i} & K_{G^yZ}^{ji} - K_{G^yZ}^{j'i} \\ \hline K_{ZG^x}^{i'j''} - K_{ZG^x}^{i'j'''} & K_{ZG^y}^{i'j''} - K_{ZG^y}^{i'j'''} & K_Z^{jj''} - K_Z^{jj'''} - K_Z^{j'j''} + K_Z^{j'j'''} \end{array}\right)$$

Then we have to write the matrix associated with the universality conditions. In the case of a second order drift it is :

$$F = \begin{pmatrix} 1 & 0 & x_j - x_{j'} \\ 0 & 1 & y_j - y_{j'} \\ 2x_j & 0 & x_j^2 - x_{j'}^2 \\ 0 & 2y_j & y_j^2 - y_{j'}^2 \\ y_j & x_j & x_j y_j - x_{j'} y_{j'} \end{pmatrix}$$

The coefficients a_i, b_i, $c_{jj'}$ and d_l of the dual kriging system can then be obtained from the kriging system:

$$\left(\begin{array}{c|c} K & F^t \\ \hline F & 0 \end{array}\right) \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} G_i^x \\ G_i^y \\ 0 \\ 0 \end{pmatrix}$$

If a unique neighborhood is used this system is solved only once to get the explicit expression of the interpolator over the whole domain.

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