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TECHNICAL PARAMETRIZATION OF ORE RESERVES

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0 - <u>INTRODUCTION</u>

Technical parametrization of recoverable ore reserves is intended as a tool for choosing among various technically feasible projects, that which has the greatest chance to lead to the best results under given economical conditions.

Each project being considered as a function of one or two purely technical parameters (e.g. cut-off grades, or tonnage of ore to be processed, or tonnage of waste to be removed), the technical parametrization consists of selecting a family of such projects which are optimal in a purely technical sense(i.e. independently of any economic consideration). This family will have to possess the following property : whatever the economic conditions of the moment (price of metals, costs of extraction and processing etc..), and the formula to be used for the economic value, the <u>economically optimal project</u> (whatever it may be) must <u>necessarily</u> fall within this family of <u>technically optimal projects</u>.

This separation (or preliminary technical parametrization) is not always possible to make rigourously, but it is often possible as a first approximation; In what follows, we will place ourselves in the simple case in which the economic value of a project is a function of the form

$$B(Q,T,V)$$

- Q : Quantity of metal (or of useful material)
- T : Tonnage of ores to be processed
- V : Total tonnage to be extracted (ore + waste)

In order to parametrize the ore reserves, we do not need to explicitly know the function B (which can thus, at this stage, remain indeterminate). We only assume the following (reasonable) hypothesis that <u>B is an increasing function of Q and a decreasing function of T and V</u>.

At that point, technical parametrization becomes possible. Among all of the possible projects involving the same tonnage T of ore to be processed and the same volume V to be mined, the best will always be the one which maximizes the quantity of recovered metal Q.

As a function of T and V, we will thus define the project which maximizes Q at fixed T and V, and we will set down the corresponding function :

$$Q = Q(T,V)$$

which will produce the parametrization which we are looking for. To choose the economically optimal project, we have only to determine (once the payable function B is known) the values of T and V which produce the maximum of $B[Q(T,V),\ T,\ V]$.

In practice, we will often replace T and V by two other parameters z and λ (representing the cut-off grades), and we will set up a parametric representation of the family of technical optima of the form :

$$Q = Q(z,\lambda)$$
$$T = T(z,\lambda)$$
$$V = V(z,\lambda)$$

The economic optimization will then (once B is known) consist of choosing the values of z and λ which maximize the expression :

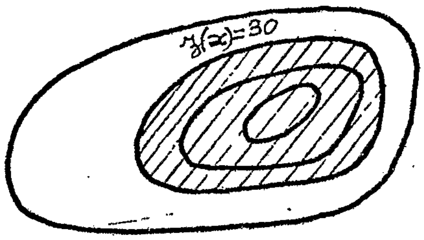
$$B[Q(z,\lambda),\ T(z,\lambda),\ V(z,\lambda)]$$

I - <u>INTRODUCTIVE INTUITION</u> : <u>ZONEOGRAPHY AND HISTOGRAM</u>.

<u>Lasky's Tonnage/Grade curve</u>.

Around 1950, S. Lasky wondered how the recoverable reserves of porphyry copper varied as a function of the usual selection criterion (the cut-off grade in Cu %). He came up with the capital notion of grade/tonnage curve.

Lasky started from a simple representation of his porphyry copper deposit : the grade Z(x) at point x was (implicitly) considered as a continuous and regular function which, for example, decreased regularly as one moved away from the rich heart of the porphyry (today, we would attribute this behaviour to a <u>trend</u>, rather than to the grade itself). He assumed this function to be known (perfectly, or to a very close approximation), and his point of view was that of a <u>zoneography</u> :



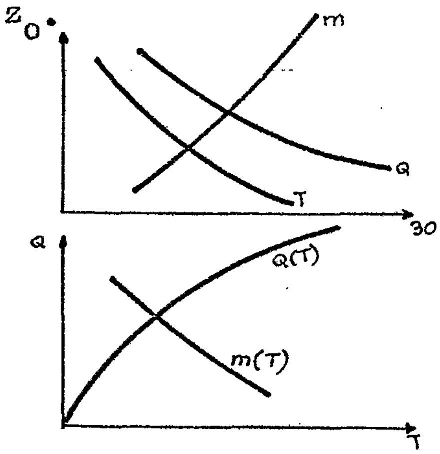
If the cut-off grade of Cu is z_o, we will mine the domain whose limits are the iso-grade z_o, i.e.

$$B_{z_o} = \{x : z(x) \geq z_o\}$$

and with a tonnage and a grade of

$$(1\text{-}1) \qquad T(z_o) = \int_{B_{z_o}} \lambda(dx) \qquad Q(z_o) = \int_{B_{z_o}} Z(x)\, \lambda(dx)$$

with : $\lambda(dx) = dx$, or, more precisely; $\rho(x)\, dx$, where ρ is the density. Lasky obtained T, Q, and $m = Q/T$ curves parametrized by z_o.



Q and T are evidently decreasing (and m increasing) with z_o.

We can thus eliminate z_o and set

$$Q = Q(T) \ , \ m = m(T), \text{ etc...}$$

[Ex : the so-called Lasky law of the form $m = \alpha - \beta \log T$].

$Q(T)$ is increasing and <u>concave</u>.

Indeed, if $T(dz)$ is the tonnage which lies between the iso-grades z and $z + dz$:

$$(1\text{-}2) \qquad \begin{aligned} T(z_o) &= \int_{z_o}^{\infty} T(dz) \\ Q(z_o) &= \int_{z_o}^{\infty} z\, T(dz) \end{aligned}$$

As a consequence, $\frac{dQ}{dT} = z_o > 0$; but $T(z_o)$ is a decreasing function of z_o, hance z_o is also a decreasing function of T : the slope of the tangeant decreases as T increases, and Q is concave.

Notice the relation :

$$Q(z_o) = z_o\, T(z_o) + \int_{z_o}^{\infty} T(z)\, dz$$

Hence :

$$m(T) = Q/T = z_o + \frac{1}{T} \int_{z_o}^{\infty} T(z)\, dz$$

The relation $Q = mT$ than gives $dQ = mdT + Tdm$, hence $z_o\, dT = mdT + Tdm$. From which we deduce that :

$$\frac{dm}{dT} = -\frac{m-z_o}{T} \leq 0$$

Thus, m decreases when T increases.

We can also give another interpretation, in terms of histograms, or of the distribution $F(dz)$ of the grades (point grades $Z(x)$). Indeed, if T_o is the maximum tonnage of ore (the limit of $T(z_o)$ as $z_o \to 0$), we may set

$$F(dz) = \frac{-1}{T_o}\, T(dz)$$

from which we obtain :

$$T(z_o) = T_o\, (1 - F(z_o)) = T_o \int_{z_o}^{\infty} F(dz)$$

$$Q(z_o) = T_o \int_{z_o}^{\infty} z\, F(dz) = z_o\, T(z_o) + T_o \int_{z_o}^{\infty} [1 - F(z)]dz$$

In this very simple case, the above formulae give the technical parametrization of recoverable reserves, as a function of a single parameter : the cut-off grade z_o.

In fact - if we suppose that the point grades are actually known and that it is possible to perform such a precise selection - the best possible choice of the volume to mine, in order to obtain a given tonnage T, would be that which maximizes Q, that is, the set $\{Z(x) \geq z_o\}$ with a z_o such that $T(z_o) = T$.

REMARK : From the point of view of geostatistics, Lasky's simplistic model is misleading. Through the (non-realistic) hypotheses made on the regionalized variable (RV) $Z(x)$: a well-known and regular function - three notions - between which one should discriminate very carefully - here become confused :

~ Zoneography
~ Histogram of grades
~ The tonnage/grade curve

With usual RV's, this method would lead to serious overestimations. Indeed, one never has the right to use the histogram of point (or core) grades ; at most, one can get away with using the histogram of grades of blocks, i.e. of the smallest units on which the effective selection will be made (their size depends upon the planned mode of operation). Then arises a first problem (which we know how to solve) :

- how to predict the histogram of blocks knowing that of the cores.

But that is not all. For the identification of the tonnage/grade curve with the distribution of blocks to be legitimate, we would need :

- the real grades to be perfectly known, at very least when the operation starts.
- that no geometric constraints should influence the selection (in other words, that we would be free to select each of the blocks independently of the other blocks).

The first condition is never achieved except (approximately) for Uranium, for which one can measure the radioactivity at the time of mining. In all other cases, the grade of the blocks will never be known exactly at the mining stage, but only estimated : based upon information perhaps richer than that which we have today, but with a margin of error nonetheless.

As for the second condition, it will rarely (or never) be possible, in practice, to extract a rich block from just any place while leaving its poorer neighbouring blocks in place. In the case of an open pit, such geometric constraints become the most important aspect of the problem.

2 - CRITICAL FACTORS OF THE TECHNICAL PARAMETRIZATION.

These factors are suggested by the preceding critique. Let us point out right away that there will be as many technical parametrizations, hence as many possible definitions of the recoverable reserves, as there are possible modes of mining. For a given mining technique, the following factors must be taken into account :

a) Geometrical constraints.

[Principle : not everything is possible].

The technological characteristics of a mining technique often impose severe constraints upon the geometrical form of the volumes to be mined. This can be expressed with the notion of admissible contours. Example :

- Open pit : The excavation must respect the natural equilibrium slope. If ore is extracted at a point x, all material within the cone T_x, which has its tip at point x (upside down) must also be extracted, and the admissible contours are necessarily a union of such cones.

- In subhorizontal bedded deposits, selection of the footwall and hanging wall limits. We decide to take only one layer of thickness greater than p. meters.

- The selection itself may be simple or it may operate at several levels. For example, we often have to perform a double selection which comprises :

~ an extraction selection concerning the choice of the volume V to be extracted (which must be an admissible contour).

~ an <u>ore selection</u> (each of the blocks v contained within the volume V to be éxtracted can be sent either to the waste or to the mill.

The geometric constraints influence each of these selections in different ways. For example, the selection of ores, at the level of the elementary blocks v, will most often be independent of any constraints (each small block, once it has been extracted, can be sent to the waste or to the mill, independently of the other blocks). On the contrary, the extraction selection will generally fall under more or less severe constraints ("the volume V to be extracted must constitute an admissible contour"). Sometimes (for example, when considering underground mining or in the case of a shallow stratified formation) we may consider (as a first approximation) that the volume V to be extracted is constituted by the union of <u>panels</u> V_j of a given size which we can choose independently of each other (this terminology implies that the blocks v, at whose level the ore is selected, are smaller than the <u>panels</u> V_j to be extracted) : in this case, we can talk about a free selection on two levels (level v for the ore selection, level V_j for the extraction selection).

b) <u>Support</u>.

[Principle : the variables upon which the ore will be mined out are not the same as those which we used in the exploration. <u>We will mine blocks or panels</u>, <u>but never cores</u>].

For each of these selections, we must defined its support, which is the size of the smallest unit which might be selected : <u>blocks</u> v for the ore selection ; <u>panels</u> V_j for the nextraction selection.

c) <u>Information</u>.

The information upon which the selection decisions are (or will be) effectively made at the various levels (ore and extraction)

[Principle : <u>the estimation is never identical to reality</u>].

We must carefully distinguish between :

- The present information (available at the moment of the parametrization of reserves : for example, exploration DDH).

- Future or "ultimate" information, which will be available when the final decision about selection is made, but generally is not available today. For example, for the selection of ores, we will have the results of the blast holes when we will have to decide whather a block v should be sent to the mill or to the waste.

This gap between present information and future information is the source of one of the principal difficulties in parametrization : since the actual future selection is based upon a not-yet-available future information, we cannot predict its effect without anticipating, probabilistically, that future information on the basis of our present information.

This would not be very serious if there are no constraints (if the panels can be selected independently), since a global probabilistic reconstitution would be possible. If there are strong constraints (open pits, for example) the prediction necessarily shifts to the local level. It then requires more powerful tools (transfer functions) and the use of the final information.

d) Criteria.

The criteria used for each of the selections. In all generality a selection S_i (for instance S_1 = extraction selection, S_2 = ore selection) requires a criterion of the form f_i (function of information available at the moment of making the effective decision upon S_i) $\geq z_i$.

These z_i constitute the parameters of the technical parametrization of reserves. Hence, in principle, there will be as many parameters as there are levels of selection. The criteria functions f_i (which we chose ahead of time) will have to respect, in principle, the following condition :

- Whatever the values of the economic parameters in the "pay off" formula itself (as long as it remains plausible), the economic optimum must belong to the family of projects defined by the criteria f_i .

3 - FREE AND SIMPLE SELECTION.

Here, the deposit is cut up into blocks (or panels) v_i and we can select each of these v_i $(i = 1,\ldots,N)$ independently. Among all of the possible choices of k blocks, the best would be that which maximizes the total recovered metal $v \sum Z(v_i)$ (this is true, regardless of the form of the pay-off formula). But we do not know the actual grades $Z(v_i)$ and we will have only their estimators $Z^*(v_i)$.

We will assume, essentially, that these estimators satisfy the conditional non-bias condition.

$$E[Z(v_i)/Z^*(v_i)] = Z^*(v_i) \tag{3-1}$$

At the present time, the estimators $Z^*(v_i)$ are not yet known, if we have available (for example) only the results of the widely-spaced exploration DDH (which gave Z_α, $\alpha = 1,\ldots,n$ as results).

For a given panel v_i, we should calculate

$$P(Z^*(v_i) \geq Z/Z_\alpha)$$

which is the probability (conditional given Z_α) thatbthe panel will be selected in the end, and estimate $T(Z)$ by

$$T(Z) = v \sum_i P(Z^*(v_i) \geq Z/Z_\alpha)$$

From this point of view, we estimate the global $T(Z)$ by summing the local estimations of each panel v_i. This is a rather heavy procedure which can be avoided here. It so happens that (if the panels are numerous enough) the summation of these various conditional probabilities approximately reproduces (in the stationary and ergodic cases) the non-conditional probability $P(Z^*(V) \geq Z)$ for a panel of size v, i.e. :

$$T(Z) = N\ v\ P(Z^*(v) \geq Z)$$

We need only to know the "a priori" distribution law of the future estimator, that is : $F^*(dZ)$.

From (3-1), $E(Z(v)/Z^*(V)) = Z^*(v)$, from which we deduce that

$$E[Z(v)/Z^*(v) \geq Z] = E[Z^*(v)/Z^*(v) \geq Z]$$

This produces our parametrization (with $T_o = N_V$) :

$$(3\text{-}2) \qquad \begin{cases} T(Z_o) = T_o \displaystyle\int_{Z_o}^{\infty} F^*(dZ) \\ Q(Z_o) = T_o \displaystyle\int_{Z_o}^{\infty} Z\ F^*(dZ) \end{cases}$$

The parametrization is similar to Lasky's (1-2) but F^* is now the distribution of the future estimator $Z^*(v)$, and not that of the block grades $Z(v)$. What's more, the second relation of (3-2) (but not the first) presupposes the condition of conditional non-bias (3-1).

In practice, we do not know the distribution F^* of the $Z^*(v)$ (nor that of the $Z(v)$), except if the future information is already available (in this case, we use the experimental histogram of the $Z^*(v_i)$. But, with the help of certain "invariance" hypotheses, we can obtain an approximate expression for F^* from the histogram of the Z_α (present information).

4 - FREE SELECTION AT TWO LEVELS.

Here, we assume that a first selection S_1 (extraction), based upon the information I_1 (not necessarily available at the time of the estimation) chooses the panels V_j which are to be extracted. Next, a second selection S_2 (of ores) chooses, within each panel, those blocks v_{ji} which are to be processed, based upon "ultimate" information I_2, which is richer than I_1, but is presently unknown. Let us assume, at the outset, that I_1 (but not I_2) is available right away.

Under these conditions, for a <u>given</u> choice of panels V_j $(j \in J)$ assumed to be selected, the reasoning followed in paragraph 3 can be applied : the economic optimum will belong to the family of projects $\{Z^*(v_{j,i}) \geq z)$ defined by the application of a single cut-off grade z to the (future) estimators $Z^*(v_{ji})$ of the various blocks contained within the preselected panels.

Thus, we should perform the same parametrization as in (3-2) but panel by panel, and consequently using the distribution law $F_j(dZ/I_1)$ of the grade $Z^*(v_j)$ of a block v_j, which sweeps through the <u>panel</u> V_j : this law is conditional, given the actual information I_1. For each panel V_j, we thus obtain a (local) estimation of recoverable reserves :

$$(4\text{-}1) \qquad \begin{cases} T_j(z_o) = V_j \displaystyle\int_{z_o}^{\infty} F_j^*\,(dZ/I_1) \\ Q_j(z_o) = V_j \displaystyle\int_{z_o}^{\infty} F_j^*\,(dz/I_1) \end{cases}$$

The techniques of <u>Disjunctive Kriging</u> are very helpful for rapidly estimating these <u>transfer functions</u> (or local parametrization of reserves), panel by panel.

We now have <u>to optimize the choise J of panels</u> V_j $(j \in J)$ to be extracted. To facilitate the reasoning (although the result which will be obtained will be more generally applicable), let us assume, temporarily, that the "pay off" formula is linear, that is, to within a constant factor :

$$B(Q,T,V) = Q - zT - \lambda V$$

(in this formula, z and λ represent the costs of processing and extraction. But we attach little importance to this economic interpretation and simply consider z and λ as two parameters which can vary independently).

For a given choice J of panels V_j to be extracted, we have

$$E(B/J) = \sum_{j \in J} [Q_j(z) - z\ T_j(z) - \lambda\ V\ j]$$

With z and λ fixed, a panel j will be selected for extraction if and only if it brings a positive contribution to the sum above. The criterion for the selection of panels is thus :

$$\frac{Q_j(z) - z\ T_j(z)}{V_j} \geq \lambda \tag{4-2}$$

If we now consider z and λ as simple variable parameters, we wi will obtain the following parametrization : for given z and λ, we will retain those panels which verify criterion (4-2), and this defines the set $J = J(z,\lambda)$ of selected panels. From this set $J(z,\lambda)$ of panels, we then select the blocks of grade $\geq$ z (ore selection). In this way, we obtain the parametrization

$$\begin{cases} Q(z,\lambda) = \sum\limits_{j \in J(z,\lambda)} Q_j(z) \\ T(z,\lambda) = \sum\limits_{j \in J(z,\lambda)} T_j(z) \\ V(z,\lambda) = \sum\limits_{j \in J(z,\lambda)} V_j \end{cases} \tag{4-3}$$

<u>REMARKS</u> :

1) Even though, for simplicity's sake, we assumed that the "pay-off" formula is linear, the parametrization (4-3) has a general value. More precisely, the techniques of <u>convex analysis</u>, which we will study, show that the projects defined by (4-3) all correspond to technical optima (for given z and λ, each of these maximizes the quantity of metal Q for fixed $T = T(z,\lambda)$ and $V = V(z,\lambda)$. However, some existing technical optima may escape this parametrization, but : those optima which elude us will always, in practice, be closely bracketed by projects defined by (4-3), and, for the

usual forms of "pay-off" formula, they would have virtually no chance of corresponding with the economic optimum.

2) We have assumed that the information I_1, for the first selection, is available. If it is not yet available, if we have only the poorer information $I_0 \subset I_1$ the $Q_j(z)$ and $T_j(z)$ must be considered as random variables, and formulae (4-3) must be probabilistically anticipated ; such a procedure is possible, but we will not go into it here.

5 - SIMPLE SELECTION UNDER CONSTRAINT.

The constraint is expressed by stating that any project must correspond to a contour $A \in \mathcal{B}$ (belonging to the family $\mathcal{B}$ of admissible contours). Examples :

- Open Pit : $A \in \mathcal{B}$ if A follows the equilibrium slope conditions, hence if A is the union of cones T_X, $X \in A$.

- Footwall and Hanging wall : Here, the deposit is divided into panels P_j. In each panel, the mined portion must be either empty or a horizontal slice of thickness $\geq$ a given p (for example). To simplify things, we assume that the horizontal slices may be independently chosen from the various panels P_j (hence, we neglect the link-up conditions between panels).

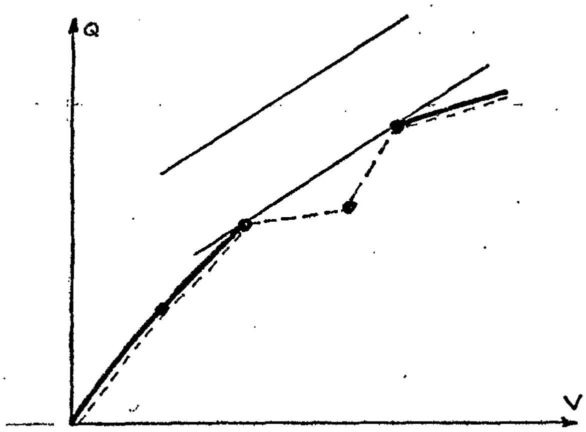
Among the possible projects A of volume $V(A) \leq$ a given v, we suppose that the best is that project which maximizes the quantity of metal $Q(A)$. For each v, there are thus one (or several) A_v which verify :

$$\text{(5-1)} \qquad \operatorname{Sup}[Q(A) : A \in \mathcal{B} \, , \, V(A) \leq v] = Q(v)$$

If the grade $q(x)$ is known at each point x, then $Q(A) = \int_A q(x)dx$ is known for each A, and the above formula will achieve (theoretically) the technical parametrization of v. When $q(x)$ is not known, nothing essential is changed if the information I, upon which is based the choice of contour A, is available right away - which is what we assume here. Indeed, we formulate, for each x, the estimator $q^*(x) = E(q(x)/I)$ which satisfies the condition of conditional non-bias. We are back to maximizing $\int_A q^*(x)dx = Q^*(A)$ (since $E(Q(A)/I) = Q^*(A)$ for each $A \in \mathcal{B}$). We shall therefore drop the asterisk and write $Q(A)$ instead of $Q^*(A)$.

Principle of Convex Analysis

In practice, the problem stated in (5-1) is rarely easy to solve. A tedious combinatorial analysis is often required. Convex analysis techniques consist then in replacing the inaccessible curve $Q = Q(v)$, which is increasing but generally not concave, by its concave hull $Q = \hat{Q}(v)$, the latter curve being in general more easy to get : doing so, we may miss a few technical optima (but they are the most interesting ones in an economical sense, and in any case they are never too far from (known) points of the concave curve $\hat{Q}(v)$.



For each $\lambda \geq 0$, let us consider with (Q,v)-coordinates the lines of equation $Q - \lambda v = \gamma$. When γ is great, the curve $Q = Q(v)$ is entirely within the lower semi-plane limited by this line : $Q(v) - \lambda v < \gamma$ for any v. For a limit value $\gamma = \gamma(\lambda)$, the line meets one, or possibly more points of the curve, but this curve is still within the lower semi-plane (i.e. $Q(v) - \lambda v \leq \gamma$). It then appears that $\gamma(\lambda)$ is given by :

(5-2) $$\gamma(\lambda) = \sup_{A \in \mathcal{B}} \{Q(A) - \lambda v(A)\}$$

and that the admissible contour(s) for which this maximum is met correspond precisely to points where the line $Q - \lambda v = \gamma(\lambda)$ meets the curve $Q = Q(v)$ (such points are called "critical points").

The concave hull $Q = \hat{Q}(v)$, whose explicit expression is

$$\hat{Q}(v) = \operatorname*{Inf}_{\lambda \geq 0} \{\gamma(\lambda) + \lambda v\}$$

is thus defined as the set of critical points of the various lines obtained when λ varies. "Missing" optimal projects correspond to convex parts of the curve $Q(v)$ and hence are, in general, of little interest. Except for those few unimportant missing projects, we have commuted the tedious problem (5-1) into a much easier problem (5-2) : find, for each value λ, the project which maximizes $Q-\lambda v$.

The recovered reserves can then be parametrized in terms of λ : for each $\lambda \geq 0$, the limit line $Q - \lambda v = \gamma(\lambda)$ presents the two extreme critical points $(v_\lambda^-, Q_\lambda^-)$ and $(v_\lambda^+, Q_\lambda^+)$, hence $v_\lambda^- \leq v_\lambda \leq v_\lambda^+$ for any other critical point), and these two extreme points correspond to the two projects A_λ^+ and A_λ^-. We have then come to the following parametrization :

$$v^+(\lambda) = v(A_\lambda^+) \qquad v^-(\lambda) = v(A_\lambda^-)$$

$$Q^+(\lambda) = Q(A_\lambda^+) \qquad Q^-(\lambda) = Q(A_\lambda^-)$$

v^+ and Q^+ are decreasing functions and continuous to the left hand side ; v^- and Q^- are decreasing and continuous to the right hand side. Moreover :

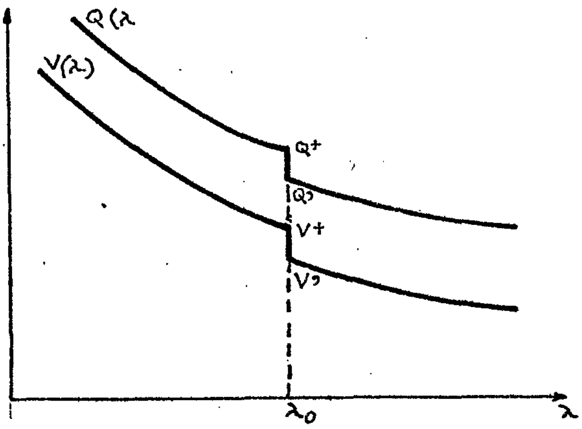
$\gamma(\lambda) = Q^+(\lambda) - \lambda\, v^+(\lambda) = Q^-(\lambda) - \lambda\, v^-(\lambda)$ is convex, decreasing and continuous.

This parametrization is unique, whatever the notations v^+, v^-, etc... because

$$v^-(\lambda) \downarrow v^+(\lambda_o) \quad , \quad Q^-(\lambda) \downarrow Q^+(\lambda_o) \quad \text{for} \quad \lambda \uparrow \lambda_o$$

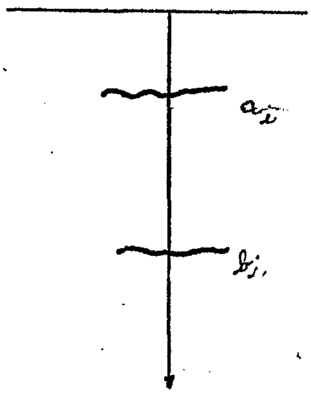
$$v^+(\lambda) \uparrow v^-(\lambda_o) \quad , \quad Q^+(\lambda) \uparrow Q^-(\lambda_o) \quad \text{for} \quad \lambda \downarrow \lambda_o$$

(this results from $v^+(\lambda) \geq v^-(\lambda) \geq v^+(\lambda_o)$ for $\lambda \uparrow \lambda_o$, etc..)



EXAMPLE 1 : Footwall and Hanging wall.

The problem is to define, for each panel P_i, the elevations a_i and b_i of the footwall and hanging wall. For each i, the kriged grade $Z_i^*(x)$ of the horizontal slice of elevation x is computed, and it follows that

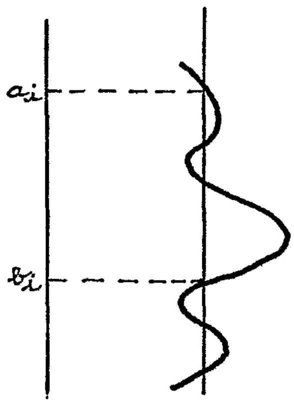


$$Q - \lambda v = \sum_i \int_{a_i}^{b_i} [Z_i^*(x) - \lambda]\, dx$$

Parametrization in terms of λ is obtained by independent optimization of the various terms :

$$\int_{a_i}^{b_i} [Z_i^*(x) - \lambda]\, dx$$

as we assumed that there was no link-up constraints between panels.



For each λ, there are a limited number of possibilities (as $Z^*(x) < \lambda$ for any x just above a_i or just below b_i). For each panel :

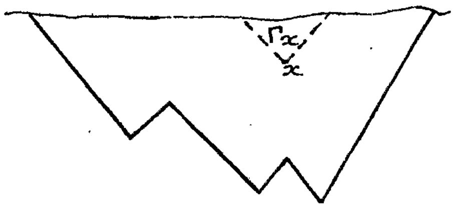
$$T_i(\lambda) = (b_i - a_i)$$

$$Q_i(\lambda) = \int_{a_i}^{b_i} Z_i^*(x)\, dx$$

then : $T(\lambda) = \sum T_i(\lambda)$; $Q(\lambda) = \sum_i Q_i(\lambda)$

EXAMPLE 2 : Open Pit design.

Here for equilibrium slope purposes, all cones Γ_x with tip at x must be mined if it is decided to mine out point x, therefore any admissible contour $A \in \mathcal{B}$ is a union of inside cones Γ_x.



It can be shown that :

1) For a given value $\lambda \geq 0$, there is a minimum size admissible contour B_λ^- and a maximum size B_λ^+ which fulfill the maximum of $Q(A) - \lambda\, v(A)$ for $A \in \mathcal{B}$, with necessarily $B_\lambda^- \subset B_\lambda^+$ (generally, $B_\lambda^- = B_\lambda^+$, except for a few critical values of parameter λ).

2) These contours are nested within each other, when λ varies :

$$B_\lambda^+ \subset B_{\lambda'}^- \quad , \quad \text{for } \lambda > \lambda'$$

3) For each point x, there exists a maximum value $\Lambda(x)$ for parameter λ, such that x belongs to the optimal contour B_λ^+ as long as $\lambda \leq \Lambda(x)$, and no longer for $\lambda > \Lambda(x)$. This function $\Lambda(x)$, when known, thus allows parametrization of the optimal contours : indeed, we have the following criterium : $x \in B_\lambda^+$ if and only if $\Lambda(x) \geq \lambda$.

4) The relation $y \in \Gamma_x$ can be considered as an ordering relationship (y is inferior to x with respect to this ordering relationship if mining x implies mining y) and a given function f is said to be increasing with respect to the order Γ if $f(y) \geq f(x)$ as soon as $y \in \Gamma_x$. Such a function $\Lambda(x)$ which performs the parametrization of optimal contours must necessarily be increasing with respect to the order Γ.

More precisely, the following result can be stated : Amongst all Γ-increasing functions, the function $\Lambda(x)$ is the one that gives the best approximation of the grade function $Z(x)$, i.e. the function minimizing the integral

$$\int [Z(x) - \Lambda(x)]^2\, dx$$

In other terms again : $\Lambda(x)$ is the <u>projection</u> of the grade $Z(x)$ on the set of Γ-increasing functions.

On this fundamental result are based procedures which allow practical parametrization of open pit recoverable reserves. The so-called "CG Open-Pit" program allows (under particular approximation) calculation of this function $\Lambda(x)$. <u>Parametrization</u> is then obtained as in the zoneography model of Lasky, replacing the grade $Z(x)$ by this function $\Lambda(x)$.

6 - <u>DOUBLE SELECTION UNDER CONSTRAINT</u>.

Admissible contours are here pairs $(A, A') \in \mathcal{B}$ where A' is the volume given by extraction selection and $A \subset A'$ is the volume given by ore selection. Let us denote $\varpi = (A, A')$; the $\varpi \in \mathcal{B}$ are then called : "<u>admissible project</u>". For any $\varpi = (A, A')$, the following notations are considered :

$V(\varpi) = V(A')$: Volume to be extracted.

$T(\varpi) = V(A)$: Volume (or tonnage) of recovered ore.

$Q(\varpi) = Q(A)$: Recovered quantity of metal.

with $V(\varpi) \geq T(\varpi)$. The general underlying idea is the duality between the two parametrizations in (V, T) and in (λ, θ).

<u>Parametrization in $(\underline{V}, \underline{T})$</u>.

For any $v \geq T \geq 0$, the function

$$Q(v, T) = \text{Sup}\ \{Q(\varpi),\ \varpi \in \mathcal{B},\ T(\varpi) \leq T,\ V(\varpi) \leq v\} \tag{6-1}$$

is an increasing function of the two variables v, T. This function provides a possible technical <u>parametrization</u> (under the only hypothesis that, between any two projects leading to the same values of v and T, the best project is the one that gives the greatest quantity of metal.)

But such a technical parametrization is, in practice, unaccessible because of the tedious combinatorial analysis involved. Here again $Q(v,T)$ should be replaced by its concave hull, which amounts to using dual parametrization.

Dual parametrization in λ, θ.

The function $Q(v,T)$ is replaced by its concave hull $\hat{Q}(v,T)$. For this, to each value of the pairs of parameters (λ,θ), we must associate (and it is generally possible to do so) the project or various projects providing the following maximum :

$$\gamma(\lambda,\theta) = \sup_{\varpi \in \mathcal{B}} \{Q(\varpi) - \lambda\, V(\varpi) - \theta\, T(\varpi)\}$$

Such projects correspond to critical points of planes $Q - \lambda V - \theta T = \gamma(\lambda,\theta)$, i.e. to the points where these planes meet the surface $Q = Q(V,T)$, this surface being within the semi-space defined by this limiting plane. Those critical points thus define the concave hull of $Q(V,T)$, i.e. :

$$\hat{Q}(V,T) = \inf_{\lambda,\theta \geq 0} \{\gamma(\lambda,\theta) + \lambda V + \theta T\}$$

and they give, as functions of λ and θ, the following technical parametrization :

$$Q = Q(\lambda,\theta) \quad ; \quad T = T(\lambda,\theta) \quad ; \quad V = V(\lambda,\theta)$$

In practice, one parameter is fixed, e.g. $\theta = \theta_o$, and the parametrization in λ is done as was shown in the preceding paragraph. Then θ_o is made variable, and the required parametrization in (λ,θ) is thus obtained.

EXAMPLE 1 : Defining footwall and hanging wall with overburden.

With the notations of section 5, it comes here :

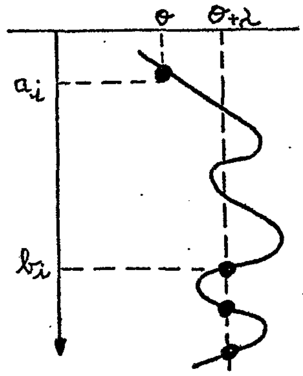
$$Q(\varpi) - \lambda\, V(\varpi) - \theta\, T(\varpi) = \sum_i \left[\int_{a_i}^{b_i} z^*(x)dx - \lambda(b_i) - \theta(b_i - a_i) \right]$$

As the limits (a_i,b_i) are determined independently from one panel to another, the following optima :

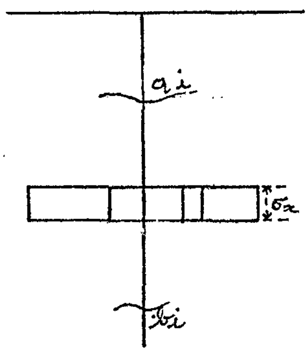
$$\int_{a_i}^{b_i} Z^*(x)dx - (\lambda+\theta)\ b_i + \theta a_i$$

must be determined independently for each panel P_i : there are but a small number of possibilities, since $Z^*(x) < \theta$ for elevation $x > a$, and $Z^*(x) \geq \theta$ for $x < a$; similarly $Z^*(x) < \lambda+\theta$ for $x < b$ and $Z^*(x) \geq \lambda+\theta$ for $x > b$. The following parametrization is thus readily obtained :

$$\begin{cases} Q(\lambda,\theta) = S \sum\limits_i \int_{a_i}^{b_i} Z^*(x)\ dx \\ T(\lambda,\theta) = S \sum\limits_i (b_i - a_i) \\ V(\lambda,\theta) = S \sum\limits_i b_i \end{cases}$$



<u>EXAMPLE 2</u> : <u>Defining footwall and hanging wall with precise selection.</u>



Let us consider an uranium deposit. Within each panel, an horizontal slice (a_i, b_i) is recovered (whatever overburden may result). Then at the mining stage a precise selection of ore is made on small blocks of thickness δx.

On these blocks v, the criterium $Z^*(v) \geq \theta$ allows the technical parametrization of a given volume (a slice a_i, b_i) already mined out. This leads to the transfer functions of each horizontal slice $(x, x+dx)$ for each panel P_i (of surface S_i)

$$\begin{cases} T_i(x;\theta) = S_i\ \delta x \int_\theta^\infty F_i^*(x;dz) \\ Q_i(x;\theta) = S_i\ \delta x \int_\theta^\infty z\ F_i^*(x;dz) \end{cases}$$

Then it follows for the slice (a_i, b_i)

$$Q_i - \lambda V_o - \theta T_i = S_i \int_{a_i}^{b_i} (Q_i(x;\theta) - \theta\ T_i(x;\theta) - \lambda)\ dx$$

For each given value θ, a_i and b_i are given by the procedure detailed in section 5 (replacing $Z^*(x)$ with $Q_i(x,\theta) - \theta\ T_i(x,\theta)$). Therefore a_i and b_i appear as functions of λ and θ, and by summing up, the parametrization $Q(\lambda,\theta)$, $T(\lambda,\theta)$, $V(\lambda,\theta)$ is obtained.

EXAMPLE 3 : Open Pit design with free selection of ore.

Each admissible contour being necessarily a union of blocks v, the problem is considered for each particular block v. For each block v_x (centred at x, $x \in E$ where E is finite), the transfer functions $T(x,\theta)$ and $Q(x,\theta)$ are computed. For each given value θ, the problem then reduces to a single λ-parametrization problem with $Z(x) = Q(x,\theta) - \theta\ T(x,\theta)$.