# Letter to the Editor

## A Simple Answer to an Elementary Question

Let a domain $V$ in an orebody be the union $V = \cup v_i$ of $N$ small units $v_i$, all disjoint and equal to a same volume $v$ (up to a translation). It seems natural to characterize the dispersion of grades $z(v_i)$ of these units $v_i$ inside the domain $V$ by the parameter

$$s^2(v \mid V) = \frac{1}{N} \sum \left[z(v_i) - z(V)\right]^2$$

Unfortunately, grades $z(v_i)$ and $z(V)$ generally are not known, and parameter $s^2(v \mid V)$ cannot be estimated without using a model. Many models are used in geostatistics. In the most common one, grades $z(x)$ (defined on a point support) are interpreted as a realization of an intrinsic random function $Z(x)$ (of order 0). Within the framework of this model, the parameter $s^2(v \mid V)$ may be interpreted as a realization of the corresponding random variable

$$S^2(v \mid V) = \frac{1}{N} \sum \left[Z(v_i) - Z(V)\right]^2$$

and the expectation of this random variable is easy to calculate with the help of the variogram $\gamma(h)$, i.e.

$$\sigma^2(v \mid V) = E\left[S^2(v \mid V)\right] = \bar{\gamma}(V, V) - \bar{\gamma}(v, v) \qquad (1)$$

In this expression, $\bar{\gamma}(V, V)$ represents the average value of $\gamma(x - y)$ when the points $x$ and $y$ are distributed uniformly inside $V$. This new parameter $\sigma^2(v \mid V)$ is called "dispersion variance" (of $v$ inside $V$). Its numerical value depends on the geometry ($v$ and $V$) and also on the choice of the variogram $\gamma(h)$. When available data allow a correct choice of the variogram, formula (1) generally gives excellent results, i.e., good agreement with the average value of $s^2(v \mid V)$ whenever it may be calculated for different domains with same size $V$ inside the same orebody.

All of this is elementary and fully explained in French: Matheron (1962, p. 57–61 and 72–73; 1965, p. 135–140); in Russian: Matheron (1968, p. 94–

100); in English: Matheron (1970, p. 66–69), David (1977, p. 97–98), Journel and Huijbregts (1978, p. 61–68), and now in many other languages.

In the particular case of a logarithmic variogram (the "de Wijsian scheme" of the sixties) and if volumes $v$ and $V$ are similar (i.e., have the same shape, but not the same size!), the general formula (1) becomes

$$\sigma^2(v \mid V) = \alpha \log (V/v) \tag{2}$$

This can be found in Matheron (1962, p. 76; 1965, p. 242; 1968, p. 98) and in David (1977, p. 180).

Concerning the original de Wijs's model, it dates back to a pregeostatistical era. It has no random function in it. Starting from a given unit $V$ with grade $Z(V) = m$, and dividing it (in mind!) into two equal parts $V/2$, de Wijs obtains two grades, say $m(1 + d)$ and $m(1 - d)$ $(0 \le d \le 1)$. By iterating this process $k$ times, and assuming (as a first approximation!) that $d$ remains constant, this results in $2^k$ small units $v$; among them

$$\binom{k}{n} \text{ have grade } (m(1 - d)^{k-n}(1 + d)^n$$

$(0 \le n \le k)$. Thus, if we choose at random one of these units $v$ among the $2^k$ existing ones, its grade $Z(v)$ is a random variable with a logbinomial distribution; in fact, we have

$$\log Z(v) = \mathit{log}\ m + k \log (1 - d) + n \log \left[(1 + d)/(1 - d)\right]$$

and $n$ is binomial. The variance of $n$ is $k/4$, and thus

$$\text{Var}\left[\log Z(v)\right] = \frac{k}{4} \log \left(\frac{1 + d}{1 - d}\right)^2$$

Moreover, $V$ being divided $2^k$ parts $v$, we have $V/v = 2^k$, i.e.

$$k = \log (V/v)/\log 2$$

It follows

$$\text{Var}\left[\log Z(v)\right] = \alpha \log (V/v) \quad \text{with}$$

$$\alpha = \frac{1}{4 \log 2}\left[\log \left(\frac{1 + d}{1 - d}\right)\right]^2 \tag{3}$$

Equations (2) and (3) have the same form. Nevertheless, the first gives the variance of the grade $Z(v)$, whereas the second concerns the variance of its logarithm.

In de Wijs's logbinomial model, the variance of $Z(v)$ is

$$\text{Var}\left[Z(v)\right] = m^2[(V/v)^{\beta} - 1] \quad \text{with}$$

$$\beta = \log (1 + d^2)/\log 2 \tag{4}$$

From the geostatistical point of view of formula (1), formula (4) corresponds to the choice of a variogram of the form

$$\gamma(h) = A\,|\,h\,|^{v} \tag{5}$$

with $v = n\beta$ if the space has $n$ dimensions. The necessary condition $0 < v < 2$ is satisfied automatically for $n = 1$ or 2, because we have $0 < \beta < 1$, but not for $n = 3$.

This connection between de Wijs's original model and geostatistics is explained in Matheron (1962, p. 308–311; 1968, p. 322–325). The interpretation of formula (4) with the help of a variogram of the form (5) is given in Matheron (1965, p. 139; 1968, p. 98).

In spite of their particular character, the "de Wijsian" equation (3) and (4) remain relatively interesting today because they express what I called a "similarity principle" (Matheron, 1962, p. 76). In more up-to-date terminology, one would speak of "fractal processes." In fact, "self-similar processes" is better because the term "fractal" is now becoming very misleading. But, whatever their names, these processes are old acquaintances for geostatisticians.

G. Matheron
*Centre de Geostatistique*
*Ecole des Mines de Paris*
*35, Rue Saint-Honore 35*
*77305 Fontainebleau, France*

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