GEOMETRIC MODELING OF A BRECCIA PIPE – COMPARING FIVE APPROACHES

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ABSTRACT

The El Teniente mine is famous not only as one of the largest known porphyry-copper ore bodies but also, among geologists, for its typical breccia pipe named “Braden”, an almost vertical poorly mineralized cone, located at the center of the mine and surrounded by early-stage mineralizations. As the edge of the pipe constitutes the limit of the deposit and of the mining operation, estimating it accurately is important.

For the ten thousand samples coded by a pipe indicator, four approaches are proposed:

Two-dimensional: among all the data, only the samples on the surface are used and the elevation of the pipe surface is estimated as a function of the northing and easting coordinates.

Binary: the samples are coded 0 or 1, and the probability of being in the pipe is estimated in three-dimensional space.

Intermediate: in the three-dimensional space, the points on the surface, initially coded 0 or 1, are replaced by 0.5 and the studied variable can then take three values: 0, 0.5 and 1.

Non-stationary: techniques of Intrinsic Random Functions of order *k* are applied on the previous variable.

In this article, the four methods are described.

The quality of the results is evaluated by cross validation and compared to the geologist approach.

The article concludes with a discussion of the relevance of using stationary techniques in such a fundamentally non-stationary context.

INTRODUCTION

As described by Skewes et al. (2002) and Maksaev et al. (2004), El Teniente is a complex multistage copper Cu-Mo ore-body with many overlapping intrusions and mineralizing events over a period of 8 million years, which creates an anomalous deposit in terms of size: it contains more than 94 million metric tons (Mt) of fine copper. The center of the deposit is composed of a late-stage diatreme known as the Braden Pipe, which is 1,200 m in diameter at the surface and close to 600 m at a depth of 1,800m. The pipe is poorly mineralized and surrounded by different kinds of mineralized geological units. Knowing the exact location of the pipe surface is important, as it constitutes the internal limit of the deposit.

The objective is to estimate the location of the pipe edge using 4,000 drill holes segmented into more than 90,000, 6 m-long, samples labeled 0 when outside the pipe and 1 when inside. Four approaches are proposed. They are evaluated by cross validation and compared to a geologist mapping of the same data modeled by hand in East-West vertical sections and level planes associated with the main infrastructures: undercut, production and haulage levels.

METHODOLOGY

Two-dimensional approach (Z as a function of x and y)

For a given drill hole, the transitions from 0 to 1 (entering the pipe) or from 1 to 0 (leaving it) are studied, situations which can occur more than once when the drill hole is on the borderline of the pipe and crosses it. This yields around 900 values of Z(x,y) (Fig. 1).

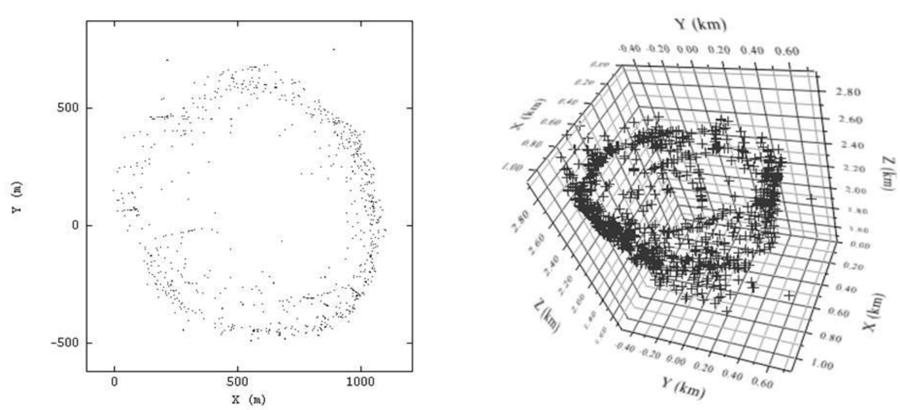


Figure 1 – Left: Location of the points on the surface; right: Elevation of these points

When only these borderline samples are used, the pipe estimation transforms into a two-dimensional interpolation of the elevation Z(x,y) that must not contain duplicates. Pairs of measurements separated by less than one meter have been removed, a crude choice easily improvable. The amount of data points is reduced to around 800. The sampling variogram was calculated as defined by Matheron (1962). It is isotropic up to 150m and modeled by a linear function of the distance plus a nugget effect (Fig. 2). The estimation is conducted by kriging (Matheron, 1963) using a unique neighborhood.

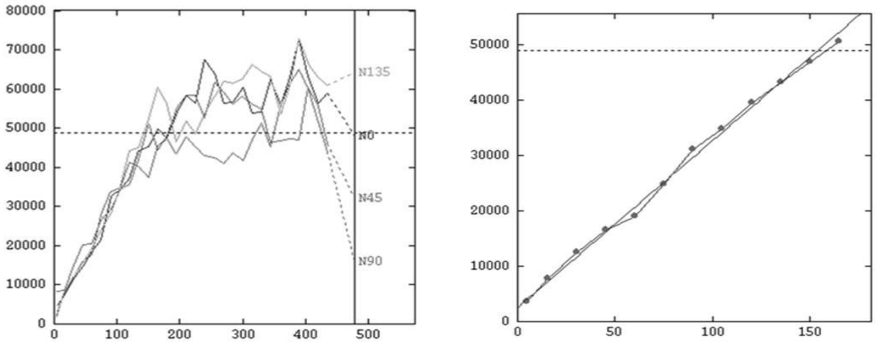


Figure 2 – Left: sampling variogram of the surface points (4 directions up to 500m); right: the model for the first 150 m (nugget + linear)

Binary approach (0 or 1)

The total number of 90,000 data labeled by 0 or 1 is used. 18% are composed by 1 (Fig. 3).

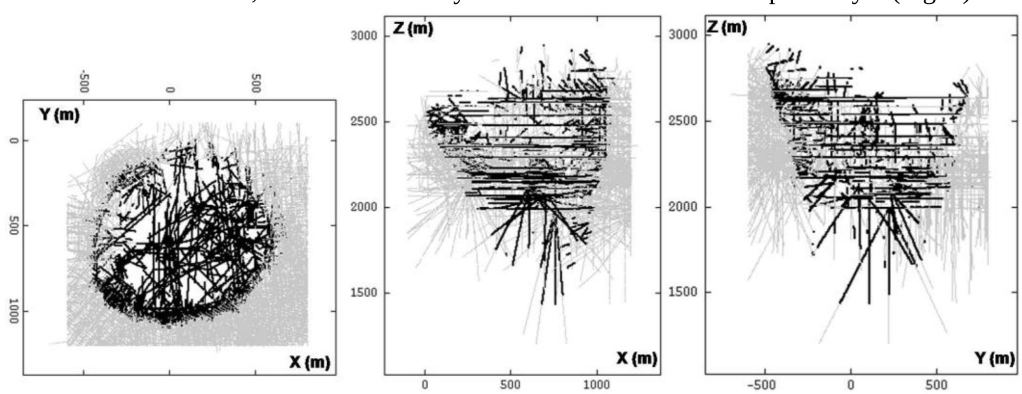


Figure 3 – Projections of the data used for the three-dimensional approach. Dark is 1, grey is 0

Let $1_{\text{pipe}}(x, y, z)$ be the function sampled in this way. Its average and variance have a probabilistic interpretation (Rivoirard 1994):

$$E[1_{\text{pipe}}(x, y, z)] = P(\ (x, y, z) \in \text{pipe}\ ) = p \tag{1}$$

$$VAR[1_{\text{pipe}}(x, y, z)] = p(1-p) \leq 0.25 \tag{2}$$

The indicator variogram as defined by Serra (1982) represents the probability for a pair of points, separated by a distance *h*, to cross the pipe surface:

$$\gamma_i(\vec{h}) = P(\ (x+h_x, y+h_y, z+h_z) \notin \text{pipe}, (x, y, z) \in \text{pipe}, \ ) \tag{3}$$

Its average behavior is isotropic for distances of less than 250m (in fact it is impossible to detect any anisotropy for such bodies, see below “Discussion”). The model is the sum of a nugget effect and a 0.7 power model (Fig. 4). The estimation is carried out by kriging with a moving neighborhood (diameter 250m).

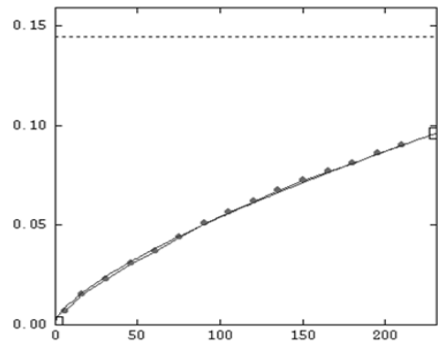


Figure 4 – Points: The sampling variogram of the pipe indicator; continuous line: The model

This approach gives a result lying between 0 and 1 in most cases. Rivoirard (1984) and Matheron (1986) have shown that some values negative or greater than 1 can appear, produced by negative kriging weights. Therefore, a normalization of the estimates to the interval [0, 1] is applied.

While this result is interpreted as a probability of belonging to the pipe, the question is: What is the value above which one must consider being inside the pipe? As the geologists and mining engineers need to map the pipe surface, they must know where the mine starts and ends. A probability does not give this result.

To answer this question, let us consider a kriging neighborhood composed of two points located at the same distance from the kriged point, one being inside the pipe, the other one outside. In the absence of anisotropy it seems reasonable to assume that in average, the target point lies on the pipe border, and in that case kriging precisely gives an estimate of 0.5. Therefore we will consider 0.5 (or 50% probability) as the key value above which we are inside the pipe

Intermediate approach (0, 0.5 or 1)

In the two-dimensional approach, around 900 points close to the surface were analyzed as functions of x and y. Now these points receive the value 0.5 (instead of the original 0 or 1) to reinforce their transition state. The variable under study can then take three values: 0, 0.5 and 1. The resulting variogram is similar to that of the previous case but for a small reduction of the nugget effect.

Non-stationary approach

The Intrinsic Random Function of order k-method (Matheron, 1973) was tested on the same dataset (values 0, 0.5 and 1). The automatic structure recognition introduced by Delfiner (1976) at the scale of the neighborhood used previously (250m diameter) yields a degree one for the drift, a result consistent with the local approximation of the pipe surface by a plane. But the automatically inferred linear variogram reveals a much steeper slope at small distances, ten times greater than the previous three-dimensional variogram (Fig. 5).

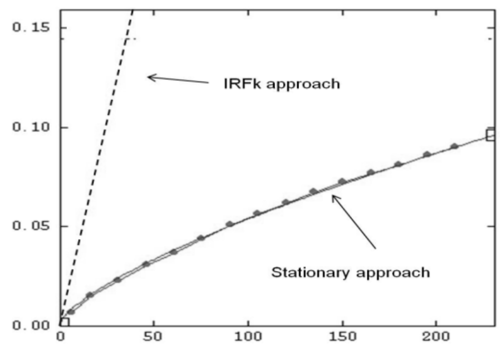


Figure 5 – Dotted line: the sampling variogram of the data used in the intermediate approach; dashed line: the linear variogram automatically inferred by the IRF-k process

RESULTS

Visual comparisons

The different approaches yield two types of results:

- The elevation of the pipe surface (two-dimensional approach);
- The probability of belonging to the pipe (other approaches).

Figure 6 (left) shows the elevation estimation given by the two-dimensional approach, and Figure 6 (right), the estimated blocks that have a probability greater than 0.5 (binary approach). The upper part of the two-dimensional elevation shows unrealistic curvature changes; the same is true for the bump at the bottom of the three-dimensional blocks. There are very few conditioning samples in these regions and the estimator extrapolates the shape. These effects are however outside the interval [1,800m, 2,700m] of interesting elevations.

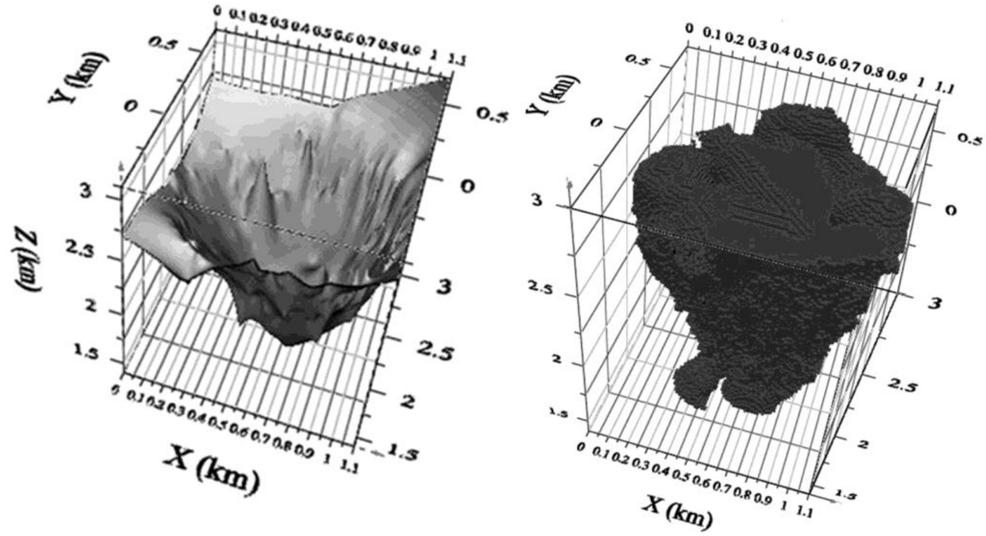


Figure 6 – Left: Elevation of the two-dimensional estimate; right: Blocks with a probability greater than 0.5

Figure 7 presents horizontal and vertical cross-sections of different estimates compared to the geologist model. The contour lines for the binary and intermediate approaches correspond to the probability 0.5. The curves are so close that one must zoom in on the areas (Fig. 8). In Figure 8, the crosses represent the points of the pipe surface set to 0.5 and one can see how they "attract" the surface.

Figure 9 presents horizontal and vertical cross-sections of the non-stationary estimate in comparison with the stationary one (binary or intermediate merged at this scale). Many artifacts appear at the bottom and the top of the pipe, they are due to the previously mentioned extrapolation effect, amplified here by the drift.

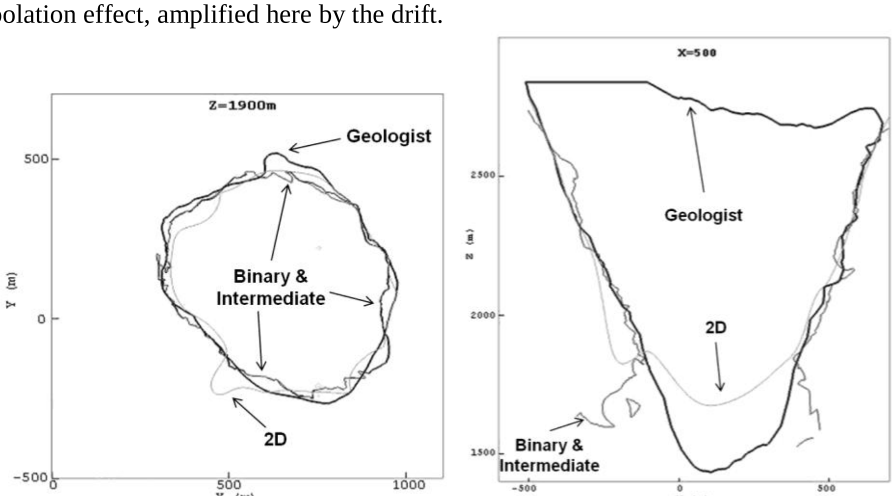


Figure 7 – Cross sections of the different approaches

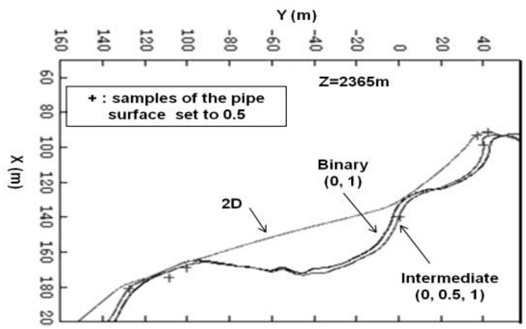


Figure 8 – Contour lines of 0.5 for binary and intermediate approaches. Crosses represent points of 0.5

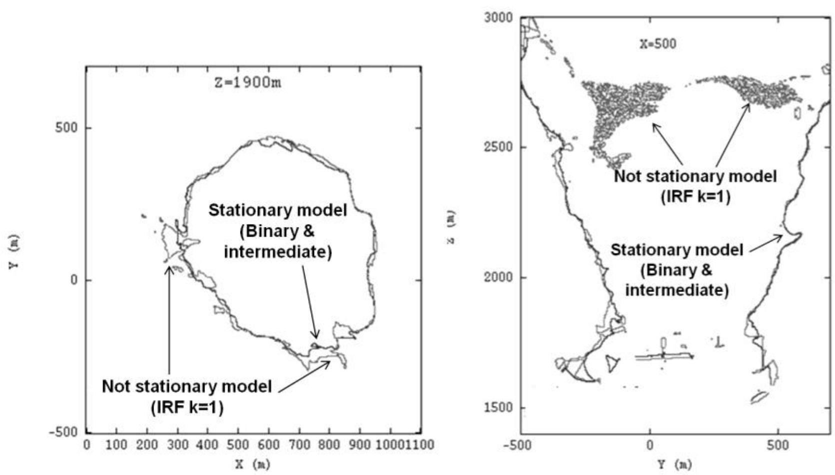


Figure 9 – Stationary versus non-stationary (binary, intermediate)

If one disregards the extrapolation problems, the first conclusion is that all the estimators are close to each other and close to the geological model. The second conclusion concerns the two-dimensional approach: only 1% of the data produces an acceptable pipe shape.

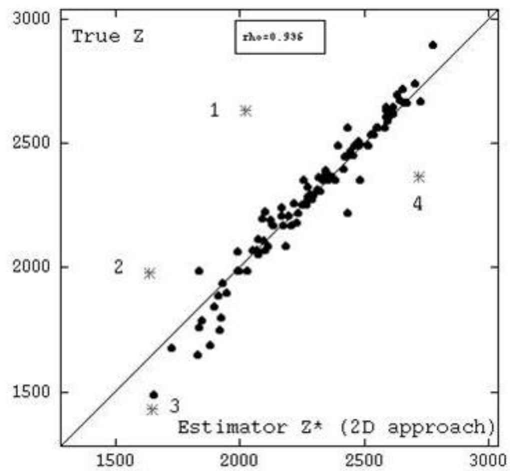
Cross validations

There are two types of results (elevation and probabilities) so there are two types of verifications:

- Among the 800 borderline samples used in the two-dimensional approach (Fig. 1), 100 are hidden and estimated using the remaining 700. The error is calculated and its histogram analyzed.

- For the other approaches (binary, intermediate, non-stationary), the probability of being in the pipe is estimated on the same borderline samples (obviously hidden during the estimation), including the duplicates removed in the two-dimensional approach (total of 919 samples). The estimation is expected to be as close as possible to 0.5.

The two-dimensional kriging generated important errors of up to 600m. Figure 10 shows the scatter diagram between the true values and estimates, as well as the locations of large errors.



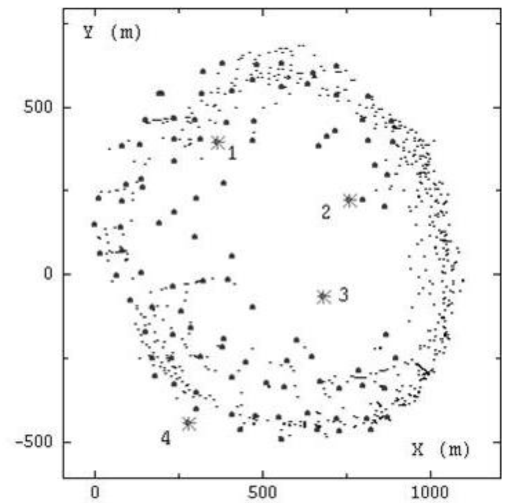


Figure 10 – Cross validation of two-dimensional results. Left: Scatter diagram between the estimator (horizontal axis) and the true measurement (vertical axis); right: Data points involved. Big dots are hidden and estimated using small dots. Points 1 to 4 are analyzed in the text

Let us analyze four of the points, labeled 1 to 4 on Fig. 10:

- Point 1 corresponds to an elevation greater than 2,700m although located close to the center of the pipe where there are mainly low elevations. The consequence is that kriging, which gives large weights to the closest surrounding points, underestimates the truth. Point 1 is probably located at the top of the pipe where the cone stops and becomes an almost flat or even hollow relief.

- Point 2 has the same problem: it is isolated, close to the center, with a true elevation equal to 2,000m whereas one finds around points with elevations of 1,600m.

- For point 3, the reverse occurs: this point is close to the center of the pipe with a true elevation close to 1,400m, probably a correct measurement considering the general behavior of the pipe. But this point is isolated and the closest samples correspond to elevations higher than 1,400 m, which conditions the overestimation;

- Point 4 belongs to the external margin of the data, where there are high elevations (around 2,700m), higher than the true one at this point (2,400m). Like point 1, point 4 is certainly a point on the surface where the relief decreases.

These four points, among others, explain some abrupt changes of the pipe shape (Fig. 7, left) and the crude underestimation of the bottom of the pipe when compared to other estimates (Fig. 7, right).

In kriging, one assumes the data to be homogeneous in each area and to evolve smoothly when moving in space. In this case, when moving to the center of the domain, a decrease in elevation is expected and the points that do not follow this implicit law become under or overestimated outliers.

Now consider the three-dimensional approaches. Figure 11 presents the histograms of estimation in binary, intermediate and non-stationary approaches applied to the surface points. As expected, the estimations are centered on 0.5. Replacing some ones and zeros by 0.5 halves the standard deviation (now 0.02). The non-stationary approach produces the same standard deviation. For the border line samples and if we neglect the artifact of the IRK-k at the top and bottom of the pipe, the stationary and non-stationary methods are equivalent when applied to the data incorporating the 0.5 values. This definitively represents an improvement.

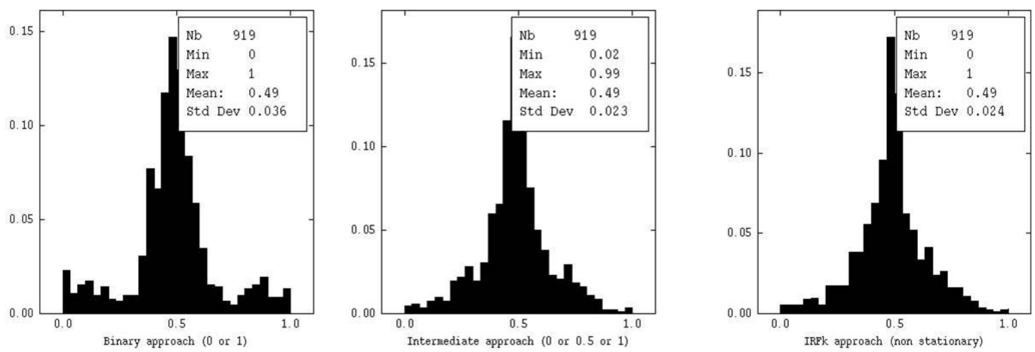


Figure 11 – Histogram of estimations for pipe surface points

DISCUSSION

There are pros and cons for each method:

- The two-dimensional approach is simple and requires less data but the duplicates need to be managed and it is very sensitive to outliers as there are few data points. Visually, the resulting estimator looks close to the geological drawings in the useful interval [1,800m, 2,700m]. The use of a unique neighborhood is certainly one cause of this proximity as it generates a smooth estimate, thus reproducing the conservative approach often followed by geologists.

- The three-dimensional approaches use all the data, can generate more details and the introduction of the label 0.5 increases the accuracy of the estimation.

- The non-stationary techniques produce dangerous artifacts at the margin of the domain and do not improve the results close to the pipe surface, so we do not recommend this approach here.

Finally, it seems better to work in three dimensions with stationary kriging applied to the data composed of 0, 1 and 0.5.

But a fundamental question remains: Is it legitimate to use stationary techniques in such a context? The two-dimensional variogram is linear (Fig. 2) whereas it should be parabolic at short distances due to the drift (Matheron, 1969).

In fact, this variogram does not characterize any spatial structure; it is just the result of a calculation applied to a particular object. Consider Figure 12 where the two-dimensional estimate is mapped (and considered to be the "truth") and imagine a pair of points separated by a distance *h*, north-south oriented, which moves all over the domain. For each pair, the difference between the elevations is squared and averaged throughout the domain to obtain the sampling variogram at the distance *h*:

- When the pairs are in the "A" areas, they cross the contour lines and measure important elevation gradients, making the variogram parabolic.
- When the pairs are in the "B" areas, they are approximately parallel to the contour lines (same elevation) and do not detect important variations, giving an almost zero value to the variogram.

The final result is a mixture of these situations and a linear behavior for distances of less than 200m.

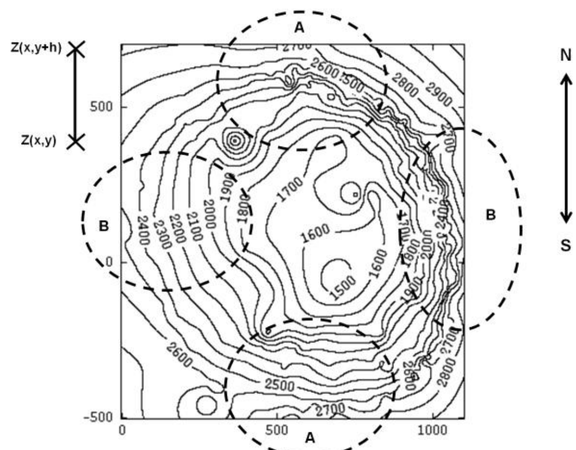


Figure 12 – Map of the pipe and domains where the N-S variogram reveals a drift (A) or do not reveal it (B)

In three-dimensions, the situation is different because the contribution of a pair of points to the variogram is bounded by 0.5, and only the pairs that cross the surface bring a nonzero value to the calculation. But the problem is that the variogram may depend on how far outside the pipe the zero sample values are and fail in capturing any structure.

Does this mean that these methods cannot be used in such circumstances? We think that they can (the results are satisfactory) and it is perhaps not necessary to follow the experimental variogram. It can be chosen according to the shape it gives to the pipe, playing the role of "shape function", a situation intermediate between a fully objective estimation and hand drawing while keeping the property of exactly fitting the data.

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