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Geostatistical Evaluation of Fracture Frequency and Crushing

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Abstract This work details how to estimate the Fracture Frequency (FF), ratio of a number of fractures divided by a sample length. The difficulty is that often, a part of the sample cannot be analyzed by the geologist because it is crushed, a characteristics of the rock strength that must also be considered for the Rock Mass Rating. After analyzing the usual practices, the paper describes the (geo)statistical link between fracturing and crushing and the resulting method to obtain a unbiased estimate of FF at a block or point support scale. Some concepts are introduced: "True" FF, "Crushed" FF, crushing probability and crushing proportion. The study is based on a real data set containing more than 13,000 samples. An appendix gives a very general formal demonstration on how to obtain an unbiased ratio estimation.

Keywords Geotechnical, Geostatistics, Mine, Fracture Frequency, Crushing, additivity, ratio estimation, RMR, FF

1 Introduction

One of the most important attribute used in the Rock Mass Rating (RMR) is the Fracture Frequency (FF), basically the ratio of a number of fractures counted by the geologist divided by the sample length. But the calculation is not that simple because it happens often that a significant part of the sample is crushed, making the fractures counting impossible, and FF becomes the ratio of two quantities which both change from a location to another one in the deposit, making difficult its evaluation, whether at sample or block scales - in other words, this ratio is not additive (Carrasco et al., 2008). To get around this difficulty, the usual practice consists in using an additive formula which combines fractures number and crush length.

The aim of this paper is:

- Analyzing the geostatistical link between fracturing and crushing,
- Proposing a unbiased way to estimate FF,
- Introducing the concept of crushing probability.

2 Formalization

Let us scheme a sample to set the vocabulary (Fig. 1).

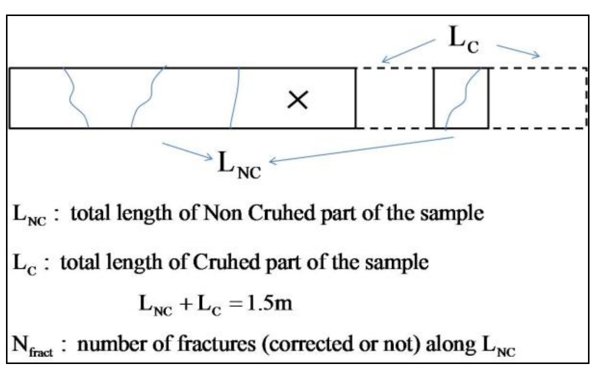

Fig. 1 Scheme presenting the useful variables, Crush Length and Fractures Number

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In the following, all the samples are supposed to have the same length (1.5 meters). For simplification, one will consider just one location "x" (center of gravity of the sample) for L_{NC}, L_C and N_{fract} . The quantities L_{NC}, L_C and N_{fract} , counted by 1.5m length, are additive and can be estimated by the basic geostatistical method called "kriging" (Matheron, 1963). N_{fract} plays the role of a "fractures accumulation", the equivalent of the "metal accumulation" in conventional mining i.e. the product of the grade by the thickness of the vein.

The quantity:

$$\mathrm{FF}_{\mathrm{true}}(x) = \frac{\mathrm{N}_{\mathrm{fract}}(x)}{\mathrm{L}_{\mathrm{NC}}(x)} \tag{1}$$

is the key frequency as it represents the true fractures frequency in the non-crushed part of the material. But it is not additive: when x moves in the space, $\mathrm{N}_{\mathrm{fract}}(x)$ and $\mathrm{L}_{\mathrm{NC}}(x)$ change and the average frequency between two measurements located at x_1 and x_2 is:

$$\mathrm{FF}_{\mathrm{true}}(x_1 \cup x_2) = \frac{\mathrm{N}_{\mathrm{fract}}(x_1) + \mathrm{N}_{\mathrm{fract}}(x_2)}{\mathrm{L}_{\mathrm{NC}}(x_1) + \mathrm{L}_{\mathrm{NC}}(x_2)}$$

This latter ratio is equal to the average of $\mathrm{FF}_{\mathrm{true}}(x_1)$ and $\mathrm{FF}_{\mathrm{true}}(x_2)$ only if $\mathrm{L}_{\mathrm{NC}}(x_1) = \mathrm{L}_{\mathrm{NC}}(x_2)$. So a direct "kriging" of $\mathrm{FF}_{\mathrm{true}}(x_0)$ for any x_0, using surrounding measurements $\mathrm{FF}_{\mathrm{true}}(x_i)$, is not possible.

This is the reason why practices consist in using the formula:

$$\mathrm{FF}_{\mathrm{corregido}}(x) = \frac{\mathrm{N}_{\mathrm{fract}}(x) + \mathrm{aL}_{\mathrm{C}}(x)}{1.5} \tag{2}$$

In (2), the coefficient "a" represents an arbitrary quantity supposed to give more or less importance to crushing in comparison with fracturing (a=40 in our case). By this way, the geotechnician incorporates the information given by crushing. (2) has also the advantage to combine additive quantities that can be estimated separately and then combined:

$$\hat{\mathrm{F}}\mathrm{F}_{\mathrm{corregido}}(x) = \frac{\mathrm{N}^*_{\mathrm{fract}}(x) + a.\mathrm{L}^*_{\mathrm{C}}(x)}{1.5} \tag{3}$$

In (3), the exponent "*" denotes various estimates.

To understand what the coefficient "a" represents, let us develop (2):

$$\mathrm{FF}_{\mathrm{corregido}}(x) = \frac{\mathrm{L}_{\mathrm{NC}}(x)\mathrm{FF}_{true}(x) + \mathrm{L}_{\mathrm{C}}(x)a}{\mathrm{L}_{\mathrm{NC}}(x) + \mathrm{L}_{\mathrm{C}}(x)} = \frac{\mathrm{L}_{\mathrm{NC}}(x)\mathrm{FF}_{true}(x) + \mathrm{L}_{\mathrm{C}}(x)\mathrm{FF}_{crushed}(x)}{\mathrm{L}_{\mathrm{NC}}(x) + \mathrm{L}_{\mathrm{C}}(x)} \tag{2'}$$

Presented in this way, (2') appears as an additive formula combining two frequencies, "a" being the one associated to crushing (now written $\mathrm{FF}_{\mathrm{crushed}}$). This latter quantity must be at least greater than any observable FF_{true} and we will detail this point in the following.

First, let us analyze the link between fracturing and crushing.

3 Observation of a natural phenomenon

We start by the examination of two samples:

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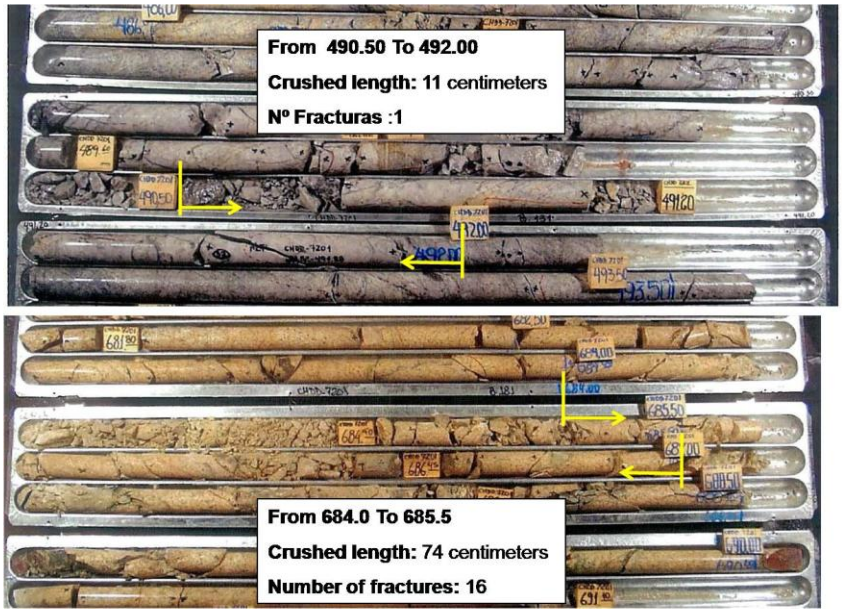

Fig. 2 Two samples (a) Few crushing and fractures (b) Important crushing, numerous fractures

Fig. 2a presents a drill core where the crush length is only 11 cm with just one fracture in the non crushed part; fig. 2b presents the contrary: crush length is important (74 cm over 1.5 m) and 16 fractures in the remaining part. Is it a particular example or is there a statistical link between N_{fract} and L_C? We have analyzed 13,000 samples (1.5 m length) coming from an underground mine in a 1000x2300x1000 m^3 box along x, y, z. (Fig. 3).

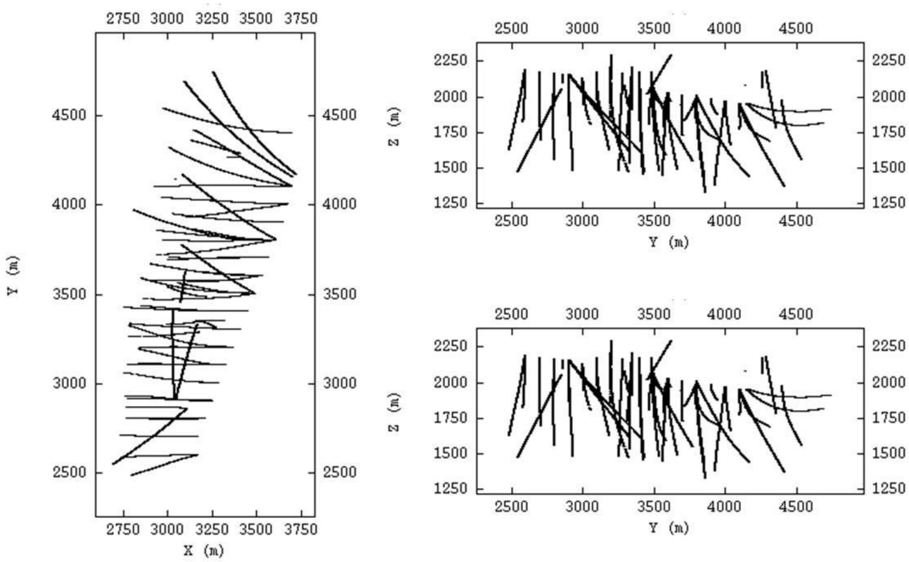

Fig. 3 Planes presenting projections of the data

The scatter diagram between N_{fract} and L_C (Fig. 4a) leads to mixed conclusions:

- The correlation coefficient is important (0.75),
- 70% of the population lies inside the confidence interval defined by the conditional expectation curve, the remaining part does not present significant correlation.

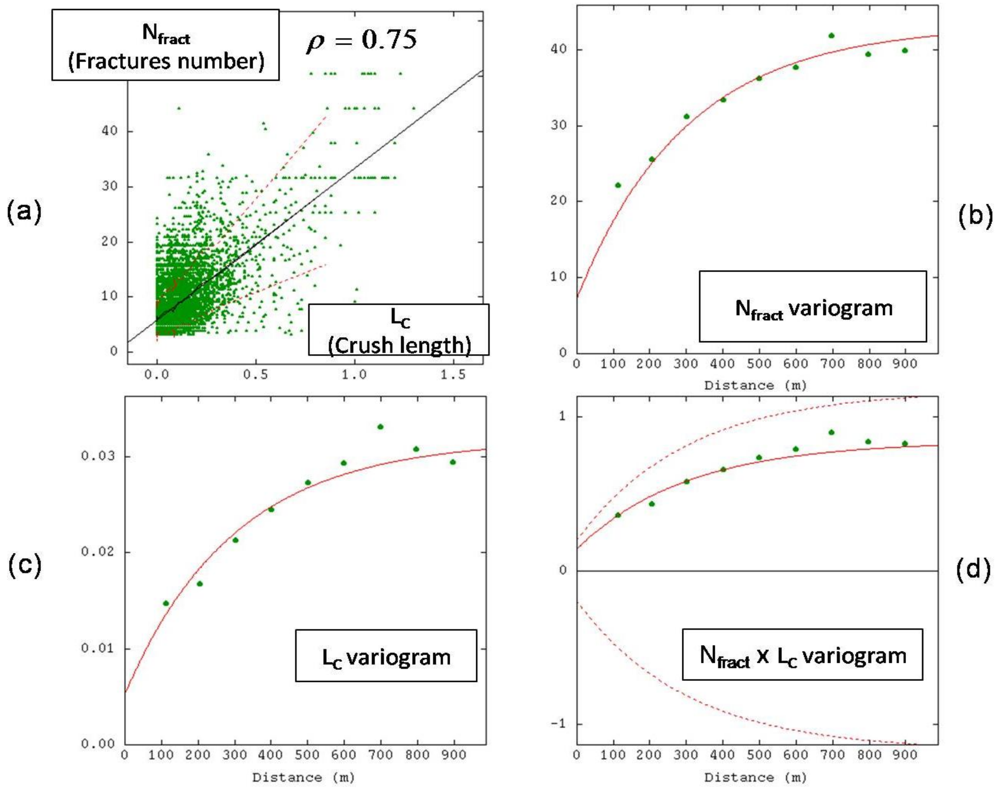

Fig. 4 (a) Scatter diagram between crush length (L_C, horizontal axis) and Fractures number (N_{fract}). Line represents the linear regression of N_{fract} against L_C, as well as the conditional expectation curve. Red dotted lines represent the standard deviation around the conditional curve. (b-c-d) Resp. N_{fract}, L_c, and N_{fract} cross L_c variograms. Points are experimental, continuous curves the intrinsic model (all the variograms are proportional)

4 True frequency estimation

Fig. 4b, 4c and 4d present respectively the direct N_{fract} variogram (Matheron, 1962, or a possible alternative calculation given by Emery et al., 2007), L_c variogram, and their cross variogram. All these variograms can be modelled by a unique model, up to a multiplicative factor – in other words, N_{fract} and L_C are in intrinsic correlation (Wackernagel, 1995).

Two important consequences result from this experimental property:

- It is not useful to use cokriging (Wackernagel, 1995) for estimating N_{fract} or L_C,
- The ratio of both estimates obtained by kriging is non biased (see Appendix)

This latter property leads immediately to the method for estimating the non additive quantity FF_{true} at a block scale V located at coordinates x:

$$FF^*_{true}(V_x) = \frac{N^K_{fract}(V_x)}{L^K_{NC}(V_x)} \qquad (4)$$

In (4), exponent K denotes the estimate of the variable by kriging, using a set of around 50 surrounding samples which change when the location x changes (”moving neighbourhood”, Chilès&Delfiner, 1999). The samples used for numerator and denominator must be the same to preserve the non bias of the ratio. In other word, we must have isotopy (Wackernagel, 1995).

Fig. 5a presents a map of $\frac{1}{FF^{*}_{true}(V_x)}$ when V_x is sized 10x10x9m^3. Geotechnicians prefer the reverse of the frequency because it represents the average size of non fractured core. When this quantity is small, the strength of the rock is bad and a low RMR rating is associated to the block. Another consequence of intrinsic correlation between both terms of the ratio is that estimating the ratio or its reverse is the same problem. Generally, this is not the case. For example, the reverse of additive grade is not additive.

5 Crushing percentage or probability

Formula (4) is a ratio of two separated estimations that can be used separately. When we divide the denominator by the sample length, we obtain an unbiased and optimal estimate of the crushing proportion:

$$P^{*}_{c}(V_x) = 1 - \frac{L^{K}_{NC}(V_x)}{\text{sample length}} \tag{5}$$

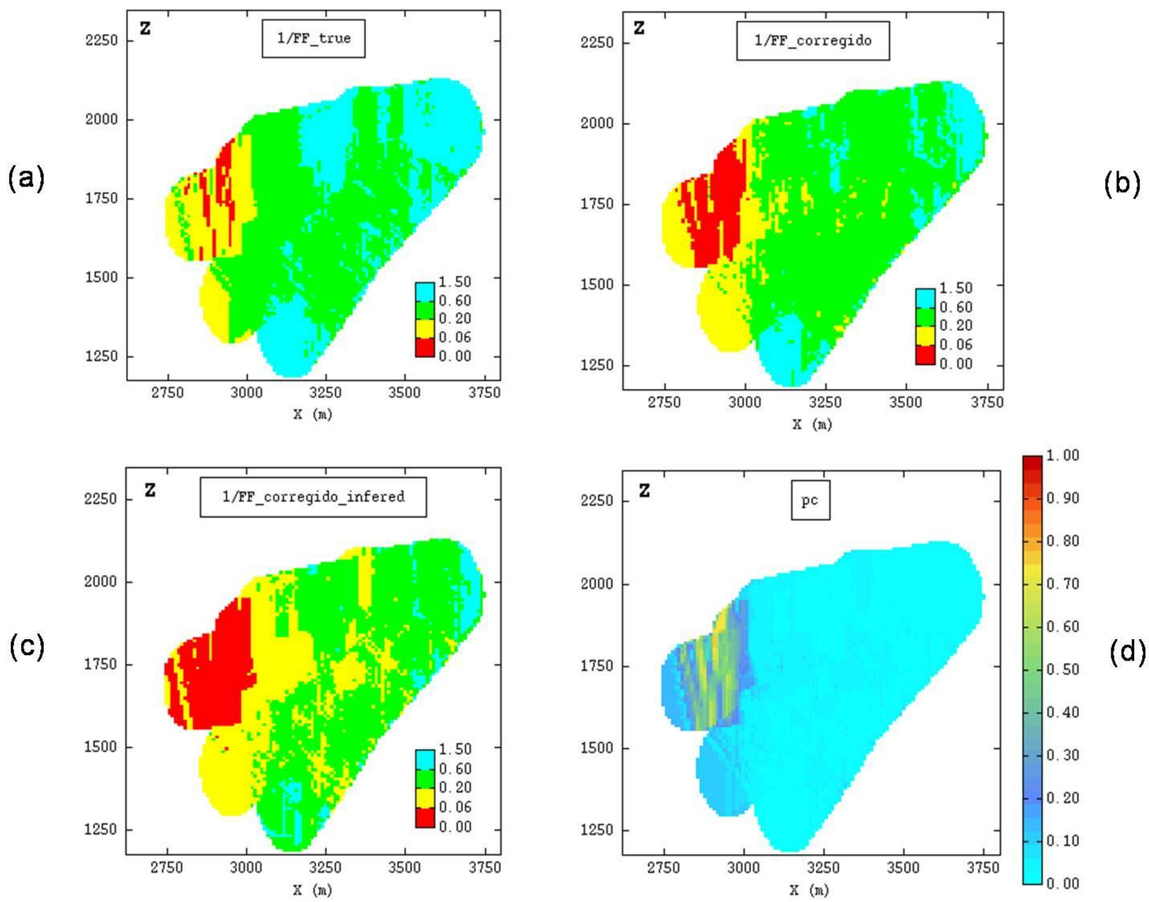

Fig. 5 (a) Map of inverse True Fracture Frequency using block kriging. (b) Map of inverse Usual Fracture Frequency that incorporates crushing estimate and arbitrary frequency for crushing equal to 40. (c) Same as (b) but with crushing frequency inferred from statistics and set to 80. (d) Crushing proportions at block scale estimated by kriging

Fig. 5d shows a cross section of the result with important crushing proportions at the West of the domain, which correspond to a well known damage zone due to a major falt.

6 Usual formulae improvement

The intrinsic correlation between crushing and fracturing leads to the optimal and unbiased estimate of formula (2) at block scale for example:

$$FF^{*}_{corregido}(V_x) = \frac{N^{K}_{fract}(V_x) + aL^{K}{}_{C}(V_x)}{1.5} \qquad (6)$$

Fig. 5b shows a cross section of $\frac{1}{FF^{*}_{corregido}(V_x)}$, a combination of fig. 5a and Fig. 5d, with the result that the West damaged zone is reinforced by accounting for crushing proportions.

7 Crushing frequency inference

Development (2') shows that the coefficient "a" used in (2) and (6) plays the role of a fracture frequency associated to crushing and named $FF_{crushed}$. In our case, for some reasons unknown when writing this paper, this quantity was set to 40 and question is: could this parameter be obtained experimentally?

Let us consider the scatter diagram between L_c and FF_{true} calculated using the 13,000 samples at our disposal (Fig. 6)

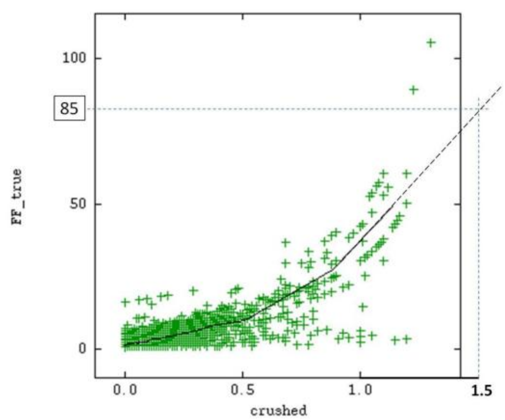

Fig. 6 Scatter diagram between crush length (L_C, horizontal axis) and FF_{true} as defined by (1). Solid line represents the conditional expectation curve; dotted segment represents a conservative extrapolation

When L_C increases, FF_{true} increases, this is a consequence of the correlation between crushing and fracturing (the number of fractures are in average more numerous when crushing length is important). The increasing rate is not linear but hyperbolic because we divide N_{fract} by a quantity which tends to zero when L_C increase.

If we suppose that:

- The crushing phenomenon appears where FF_{True} is high,
- $FF_{crushed} > FF_{true}$,
- On average $FF_{crushed}$ is independent from L_C ,

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then $FF_{crushed}$ can be characterized by its average (reference to the conditional expectation curve) and must be at least equal to the limit of FF_{True} when L_C tends to 1.5m. Fig. 6 shows that $FF_{True} = 40$ for L_C around 1m. There is still a part of the sample which is not crushed, in contrary to the previous hypothesis and $FF_{crushed}$ must be at least greater than the maximum of $E[FF_{true}|L_C]$ we can calculate, here 50 at $L_C = 1.14m$. If we make a crude linear extrapolation of the curve we obtain, for $L_C = 1.5m$

$$FF_{crushed} > FF_{true} = 85$$

As every extrapolation, this result is extremely sensitive to the hypothesis on the non linear regression modeling. The mapping of the Fracture Frequency obtained when we replace 45 by 85 in (2) is presented in Fig. 5c. Compared to the map using the traditional formula (Fig. 5 b), the West damage zone is reinforced because the influence of crushing is multiplied by more than two.

8 Conclusions

Analysis of usual practices and properties of the two variables involved in the Fracture Frequency – the Crush length and the Fracture number – does not require including both quantities in a single arbitrary formula. Analysis of a data set showed that both variables are statistically highly correlated as well as spatially and they share the same variogram. This circumstance makes possible to estimate directly the real interesting quantity which is the ratio of fractures number divided by the sample length really analyzed and shortcuts the lack of additivity of this ratio. The resulting estimate is unbiased, a basic requirement when evaluating a quantity.

On the other hand, the crushing phenomena must be estimated separately, giving a crushing proportion (at block scale) or a crushing probability (at point support scale) that must be incorporated in RMR in the same way as FF and other geotechnical attributes.

All these possibilities depend directly on the mutual behavior of Fractures number and Crush length and any study on the subject should start by the geostatistical analysis of these two variables. A more detailed analysis of their link, and another case study, which will be published in the next future, showed that the present observed correlation is not due to hazard: fracturing sometime contributes to crushing, sometime not, depending on the mutual organization of the fractures. Finally, with such studies, we evaluate the mechanical properties of the rock.

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Appendix : Unbiased Ratio Estimation

- Consider $Z_1(x)$ and $Z_2(x)$, two unknown values to be estimated using a set of *2n* measurements $\{Z_1(x_i), Z_2(x_i), i:1,n\}$. Let "*" denote any estimate and w_i any scalars.

If

$$\frac{Z_1^*(x)}{Z_2^*(x)} = \sum_{i=1}^{n} w_i \frac{Z_1(x_i)}{Z_2(x_i)} \text{ with } \sum_{i=1}^{n} w_i = 1 \quad (7)$$

then the ratio is unbiased on average if we assume its order one stationarity at the neighbourhood scale.

Proof:

$$E[\frac{Z_1^*(x)}{Z_2^*(x)}] = \sum_{i=1}^{n} w_i E[\frac{Z_1(x_i)}{Z_2(x_i)}] = E[\frac{Z_1(x)}{Z_2(x)}]\sum_{i=1}^{n} w_i = E[\frac{Z_1(x)}{Z_2(x)}]$$

- If "*" is Kriging (whether Ordinary or Simple, Rivoirard 1984), with the same variogram for Z_1 and Z_2 and same sample locations for both variables (isotopy), then the ratio is unbiased.

Proof:

As the kriging weights λ_i are identical for both terms of the ratio, we have

$$\frac{Z_1^K(x)}{Z_2^K(x)} = \frac{\sum_{i=1}^{n} \lambda_i Z_1(x_i)}{\sum_{j=1}^{n} \lambda_j Z_2(x_i)} = \frac{\sum_{i=1}^{n} \lambda_i Z_2(x_i) \frac{Z_1(x_i)}{Z_2(x_i)}}{\sum_{j=1}^{n} \lambda_j Z_2(x_j)} = \sum_{i=1}^{n} w_i \frac{Z_1(x_i)}{Z_2(x_i)}$$

with

$$w_i = \frac{\lambda_i Z_2(x_i)}{\sum_{j=1}^{n} \lambda_j Z_2(x_j)} \text{ and } \sum_{i=1}^{n} w_i = 1$$

(7) is verified, the ratio is unbiased.

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